Addendum

In the cxcxs case, if the frequencies are non-resonant, the truncated system in NF is integrable

$$\hat{H} = \hat{H}(I_1, I_2, I_3)$$
$$= \omega_1 I_1 + \omega_2 I_2 + \lambda I_3 + \hat{H}_2(I_1, I_2, I_3),$$

so that

$$\dot{I}_1 = 0,$$
$$\dot{\phi}_1 = \frac{\partial \hat{H}(I)}{\partial I_1} = \omega_1 + \frac{\partial \hat{H}_p(I)}{\partial I_1} =: \overline{\omega}_1(I),$$

$$\dot{I}_2 = 0,$$
$$\dot{\phi}_2 = \overline{\omega}_2(I),$$

$$\dot{I}_3 = 0,$$
$$\dot{s} = \frac{\partial \hat{H}(I)}{\partial I_3} = \lambda + \frac{\partial \hat{H}_p}{\partial I_3} =: \overline{\lambda}(I).$$

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