Solar Sailing near a collinear point

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Contents

What is Solar Sailing

- The impact of the photons from the Sun on the sail surface and its further reflection produces momentum on it.

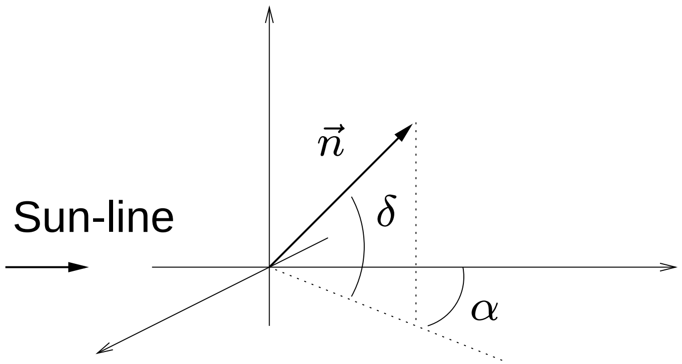
- If we have a perfectly reflecting sail,

$$\vec{F}_{Sail} = \beta \frac{m_S}{r_{PS}^2} \langle \vec{r}_{PS}, \vec{n} \rangle^2 \vec{n}.$$

- Where $\beta = \frac{L_S}{2\pi G m_S c \sigma}$ is known as the sail lightness number ($\sigma = m/A$ is the sail loading).

Sail Orientation

- The sail orientation is given by the sail normal vector ($\vec{n}$), parametrised by two angles, the pitch angle (α) and the yaw angle (δ), where $\alpha \in [-\pi/2, \pi/2]$ and $\delta \in [-\pi/2, \pi/2]$.



- The sail position with respect to the Sun can be parametrised by r, ϕ, ψ :

$$\begin{aligned} x &= r \cos\phi \cos\psi + \mu, \\ y &= r \sin\phi \cos\psi, \\ z &= r \sin\psi, \end{aligned}$$

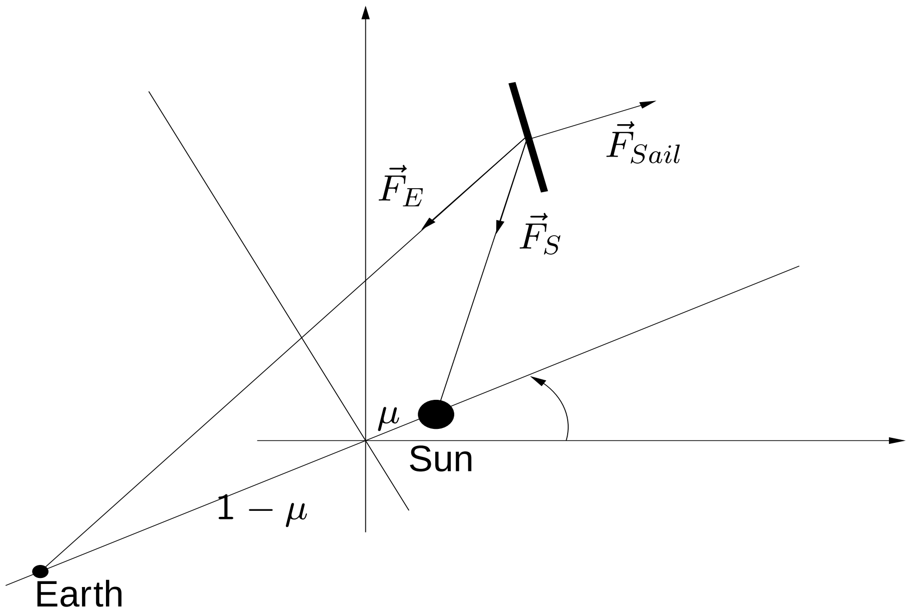
then $\vec{n} = (\cos(\phi + \alpha)\sin(\psi + \delta), \sin(\phi + \alpha)\cos(\psi + \delta), \sin(\psi + \delta))$.

Equations of Motion (RTBPS)

- Gravitational attraction from the Sun: $\vec{F}_S = (1-\mu)\frac{\vec{r}_{PS}}{r_{PS}^3}$.

- Gravitational attraction from the Earth: $\vec{F}_E = \mu\frac{\vec{r}_{PE}}{r_{PE}^3}$.

- Solar Pressure: $\vec{F}_{Sail} = \beta\frac{1-\mu}{r_{PS}^2}\langle\vec{r}_{PS},\vec{n}\rangle^2\vec{n}$.



Equations of Motion (RTBPS)

$$
\begin{aligned}
\dot{x} &= p_x + y, \qquad \dot{y} = p_y - x, \qquad \dot{z} = p_z, \\
\dot{p}_x &= p_y - (1-\mu)\frac{x-\mu}{r_{PS}^3} - \mu\frac{x+1-\mu}{r_{PT}^3} + \kappa\cos(\phi+\alpha)\cos(\psi+\delta), \\
\dot{p}_y &= -p_x - \left(\frac{1-\mu}{r_{PS}^3} + \frac{\mu}{r_{PT}^3}\right) y + \kappa\sin(\phi+\alpha)\cos(\psi+\delta), \\
\dot{p}_z &= -\left(\frac{1-\mu}{r_{PS}^3} + \frac{\mu}{r_{PT}^3}\right) z + \kappa\sin(\psi+\delta),
\end{aligned}
$$

where $\kappa = \beta\frac{1-\mu}{r_{PS}^2}\cos^2\alpha\cos^2\delta$.

If $\beta = 0$ or $\alpha = \pm\pi/2$ or $\delta = \pm\pi/2$ we have the RTBP (i.e. eliminate the sail effect).

Equilibrium Solutions

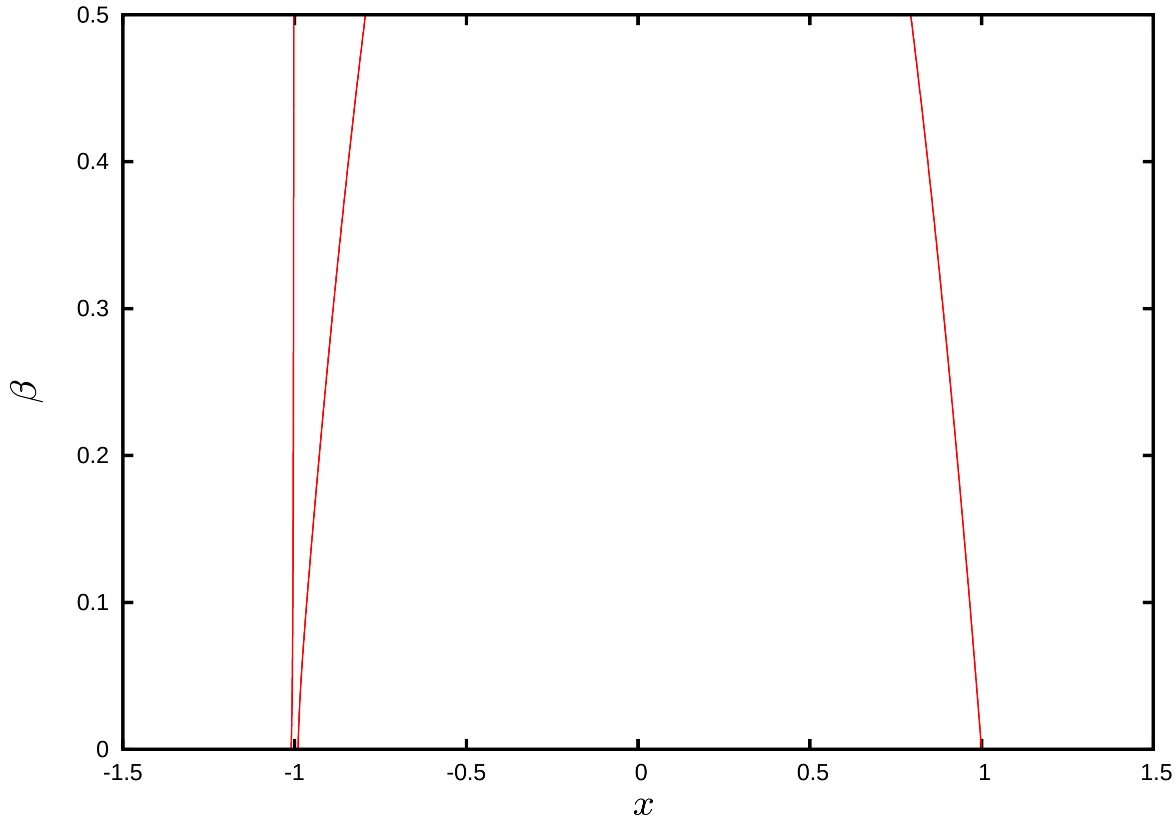
- The RTBP has 5 equilibrium points (L_i). For small β, these 5 points are replaced by 5 continuous families of equilibria, parametrised by α and δ.

- For small β, each of the surfaces of equilibria for $L_{1,2,3}$ intersect the x-axis in two points, one of them is the collinear point, the other one corresponds to $\alpha = \delta = 0$. These other points are known as Sub-$L_{1,2,3}$.

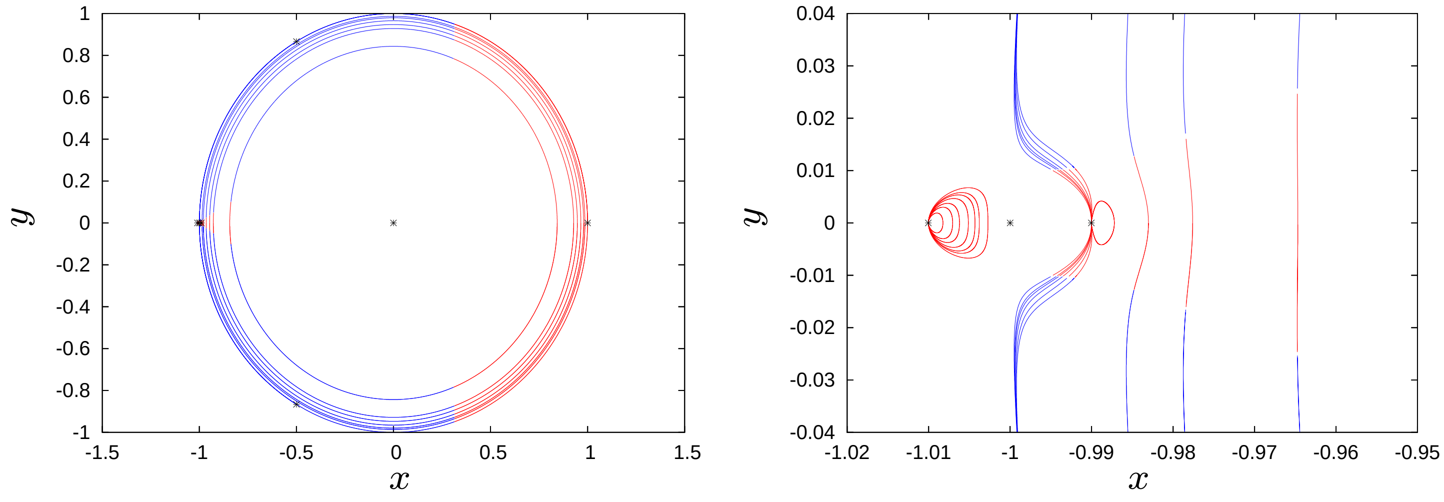
- All these families can be computed numerically by means of a continuation method.

Equilibrium solutions

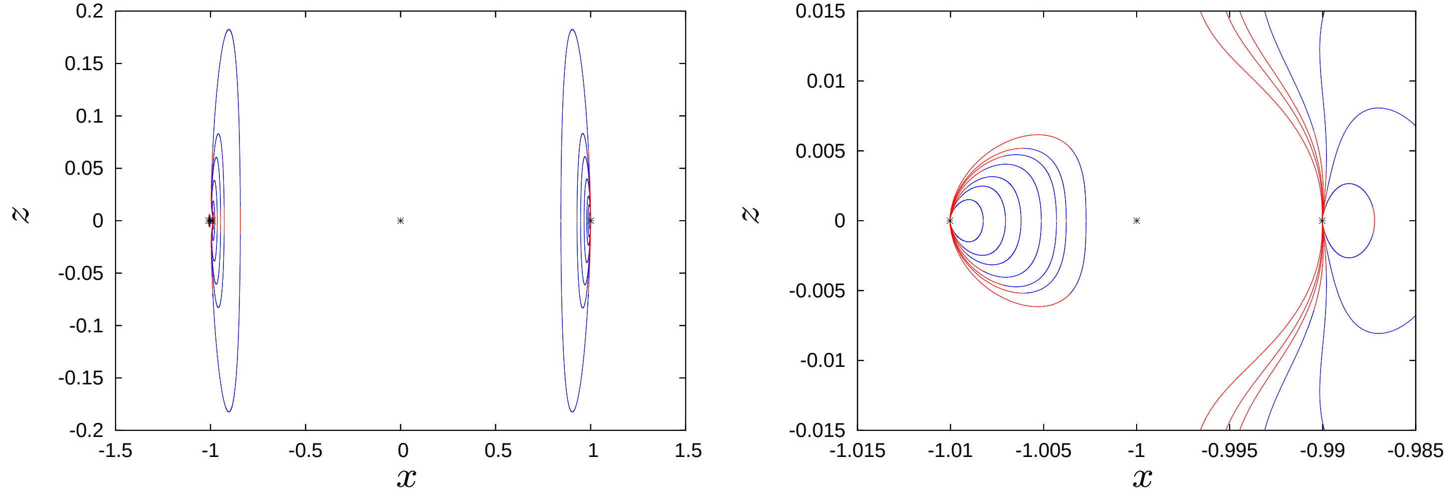
Sub-$L_{1,2,3}$ Vs $\beta \in [0, 0.5]$, remember $\alpha = \delta = 0$.



Equilibrium solutions



Equilibrium points in the $\{x, y\}$*- plane*



Equilibrium points in the $\{x, z\}$*- plane*

Possible Missions

- Observations of the Sun, as in the Geostorm mission. Observations of the Sun to provide information of the geomagnetic storms to allow for preventive actions.

- Observations of the Earth's north and south poles, situating the sail in one of the fixed points discussed before where we can observe directly the north/south pole.

- These missions are described in more detail in the book " Solar Sailing: Technology, Dynamics and Mission Applications", by Colin R. McInnes.

Our Goal

- Maintain a satellite on one of the fixed points described before.
- We have to deal with the instability of these points.
- Instead of using Control Theory Algorithm's, we will use Dynamical Systems tools to control this instability.

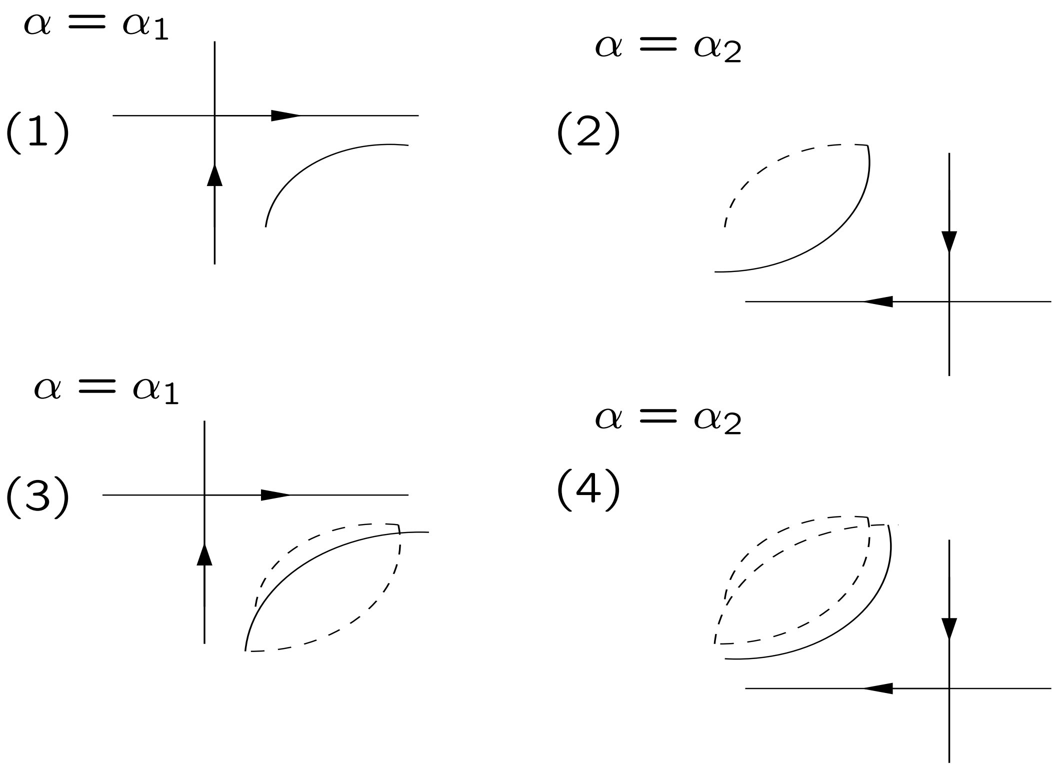
Idea:

- Change the phase space (i.e. the sail parameters) to make the system act as we wish.
- We will play with two fixed points for different parameters and their stable and unstable manifolds.

Toy Model

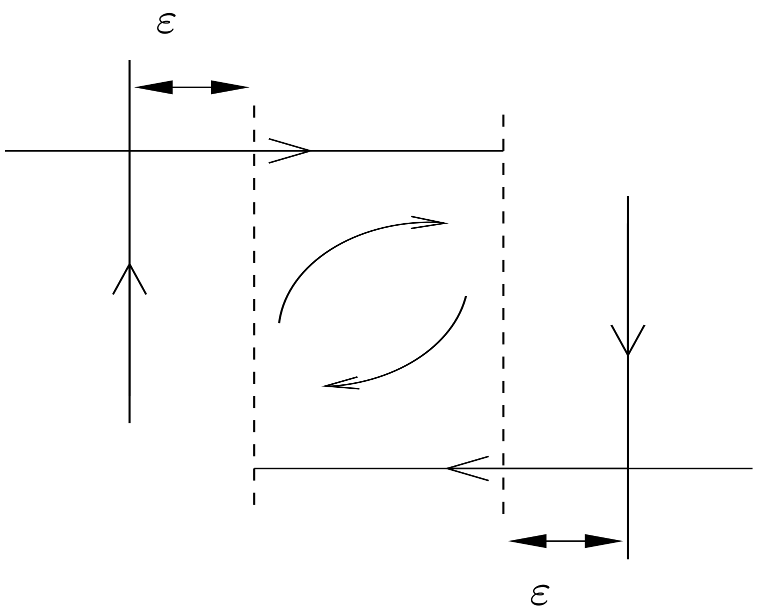
We will restrict to the linear part, where the different fixed points are saddle$\times$center$\times$center. Suppose that the eigendirections are the same for the two equilibrium points.

- We will try to control the saddle behavior varying the parameter.



Saddle Behavior

We will fix a band of width ε where we will change the parameter (i.e. the phase space).

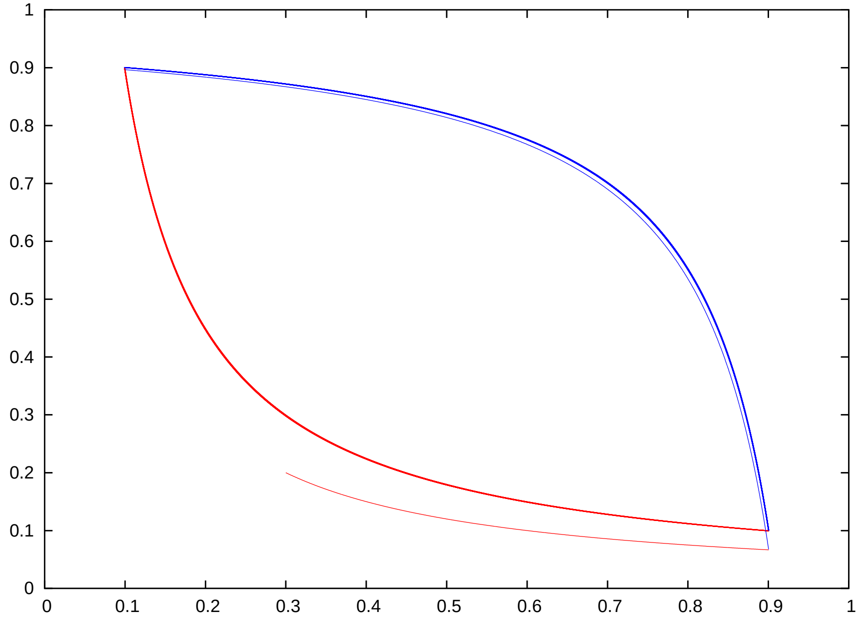


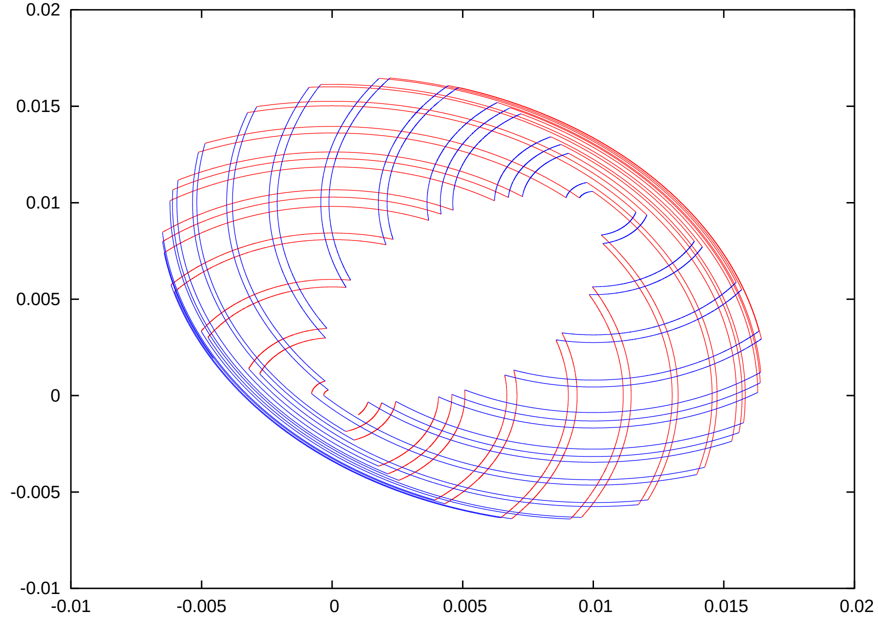
The time from one band to the other is $\Delta t_i = \frac{1}{\lambda_i} \ln(\frac{x-\varepsilon}{\varepsilon})$.

What happens with the central behavior?

Simulation

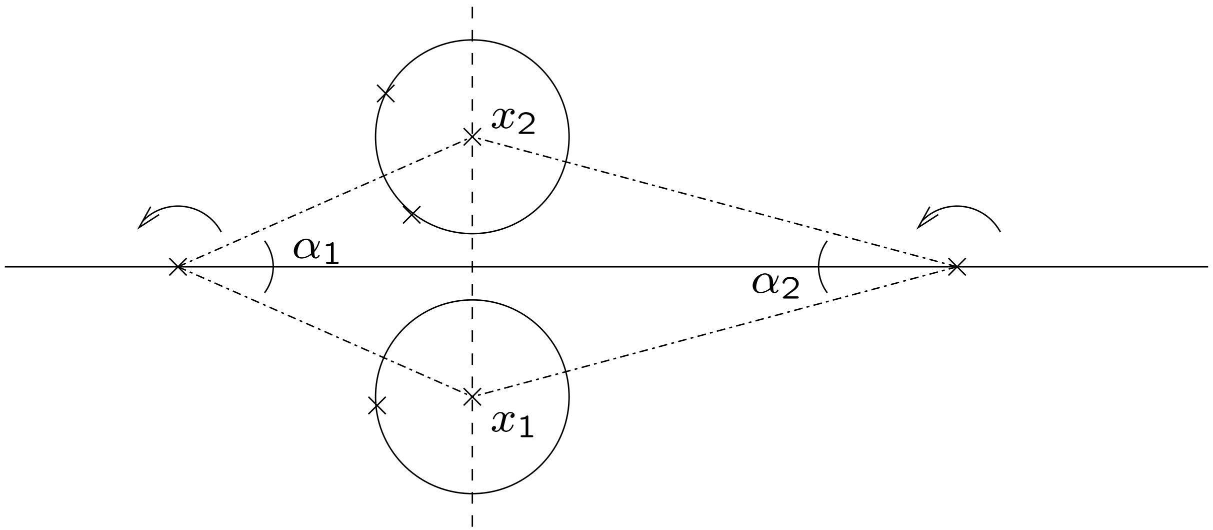
Simulation of the linear behavior:





Center Behavior

The motion is a sequence of rotations of angle $\alpha_i = \omega_i \Delta t_i$ around the two equilibrium points.



The centre behavior is bounded.

Center Behavior

- Although the projection of the motion on the center manifold is bounded we want to be able to control it.

- The growth of the central part depends on the time between manoeuvres, and the initial condition.

- It is possible to decrease the central part by playing with the time between manoeuvres.

RTBPS Model

- The stable and unstable directions are different for each of the fixed points, but if these points are close (i.e. α, δ close), the difference between them is small.

- The is also true for the center manifolds.

We would like to apply the techniques seen for the Toy Model to the RTBPS where the invariant manifolds of two fixed points are slightly different. We will design a more accurate algorithm that will take into account these differences.

Control Algorithm

Let's consider at each fixed point the reference system:

$$\{x_o; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5, \vec{v}_6\},$$

- x_o is the equilibrium point.
- $\vec{v}_1$ is the unstable direction.
- $\vec{v}_2$ is the stable direction.
- $\vec{v}_3, \vec{v}_4$ defines one of the centers.
- $\vec{v}_5, \vec{v}_6$ defines the other center.

Control Algorithm

- for $\alpha = \alpha_1$, $sist_1 = \{x_o; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5, \vec{v}_6\}$ and $z = x_o + \sum \gamma_i \vec{v}_i$.
- for $\alpha = \alpha_2$, $sist_2 = \{y_o; \vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4, \vec{u}_5, \vec{u}_6\}$ and $z = y_o + \sum \xi_i \vec{u}_i$.

The Control Algorithm:

- while $\alpha = \alpha_1$: if $|\xi_1| < \varepsilon \Rightarrow \alpha = \alpha_2$.
- while $\alpha = \alpha_2$: if $|\gamma_1| < \varepsilon \Rightarrow \alpha = \alpha_1$.

Results

Example:

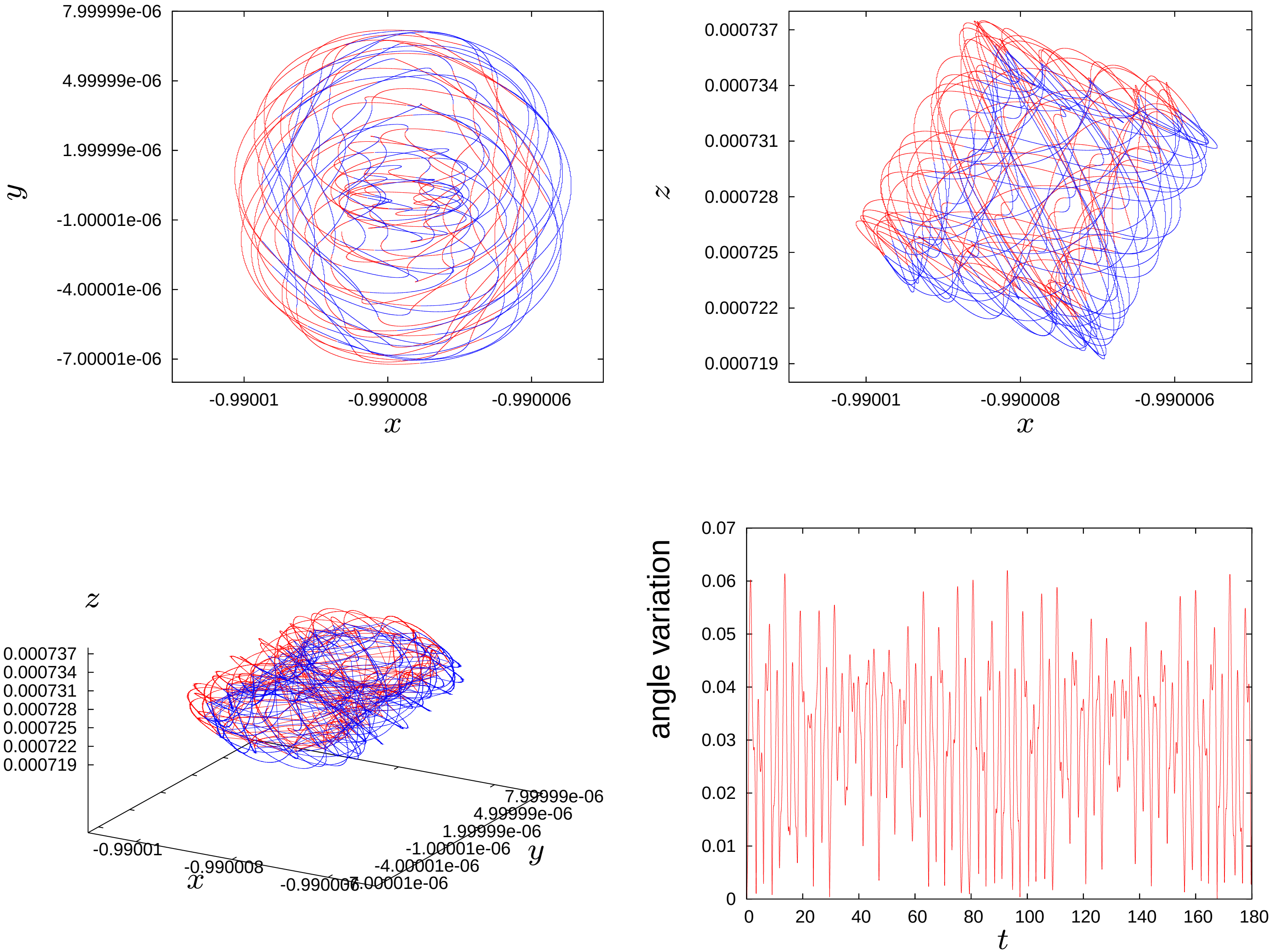
- Consider a spacecraft of mass 160 Kg and sail area 125×125 m^2 then the sail lightness number is $\beta = 0.15$.

- We have considered two points on the $\{x, z\}$ - plane at 40.8^o and 42.6^o from the x - axis.

- Distance between the points 4.685906×10^3 Km (3.132334×10^{-5} AU).

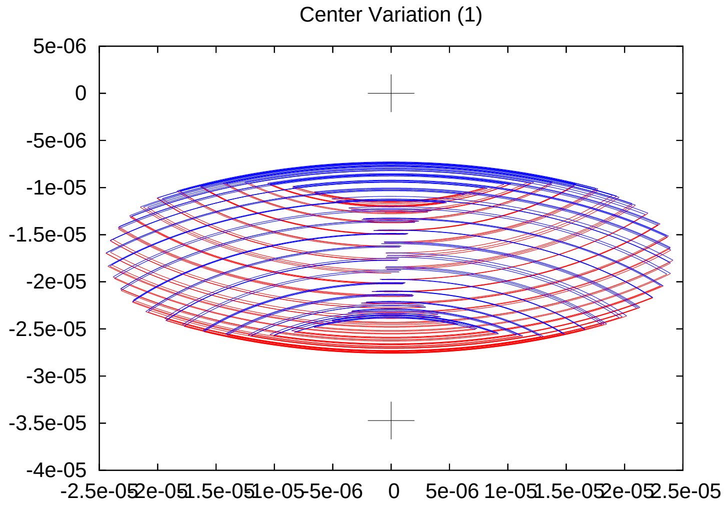
- Manoeuvres every $25 - 28$ days.

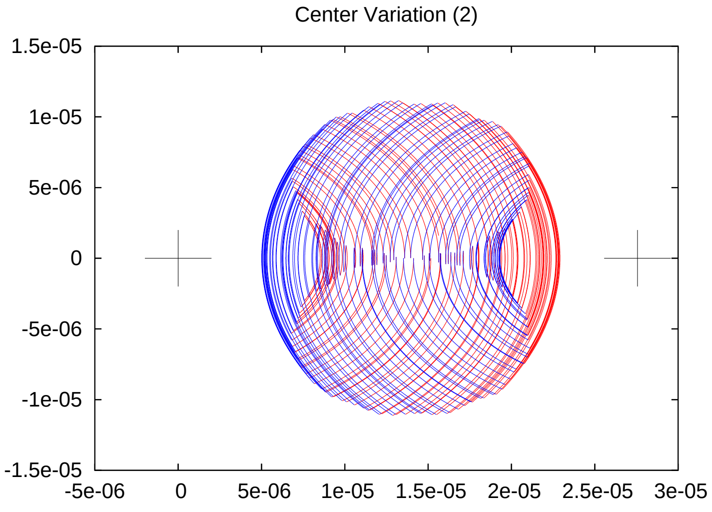
Results

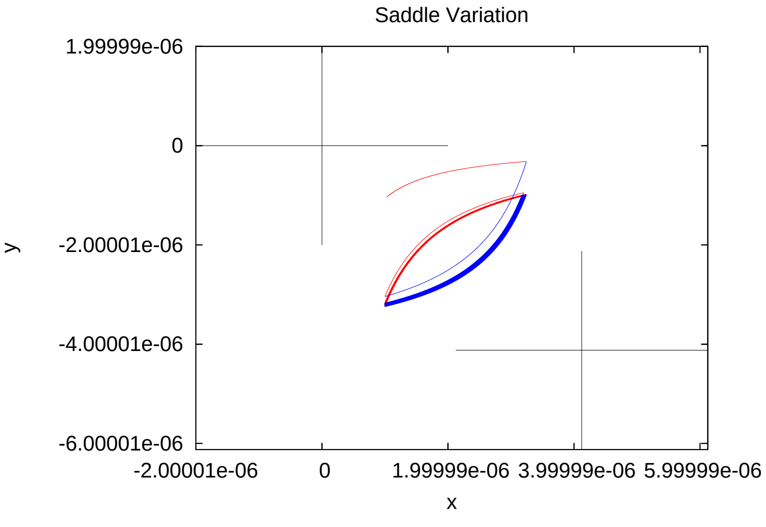


7.99999e-06
4.99999e-06
1.99999e-06
-1.00001e-06
-4.00001e-06
-7.00001e-06
y
-0.99001
-0.990008
-0.990006
x
0.000737
0.000734
0.000731
0.000728
0.000725
0.000722
0.000719
z
-0.99001
-0.990008
-0.990006
x
z
0.000737
0.000734
0.000731
0.000728
0.000725
0.000722
0.000719
-0.99001
-0.990008
-0.990006
x
7.99999e-06
4.99999e-06
1.99999e-06
-1.00001e-06
-4.00001e-06
-7.00001e-06
y
angle variation
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
20
40
60
80
100
120
140
160
180
t

Results


Center Variation (1)
5e-06
0
-5e-06
-1e-05
-1.5e-05
-2e-05
-2.5e-05
-3e-05
-3.5e-05
-4e-05
-2.5e-05
-2e-05
-1.5e-05
-1e-05
-5e-06
0
5e-06
1e-05
1.5e-05
2e-05
2.5e-05


Center Variation (2)
1.5e-05
1e-05
5e-06
0
-5e-06
-1e-05
-1.5e-05
-5e-06
0
5e-06
1e-05
1.5e-05
2e-05
2.5e-05
3e-05


Saddle Variation
1.99999e-06
0
-2.00001e-06
-4.00001e-06
-6.00001e-06
y
-2.00001e-06
0
1.99999e-06
3.99999e-06
5.99999e-06
x

Results

Example:

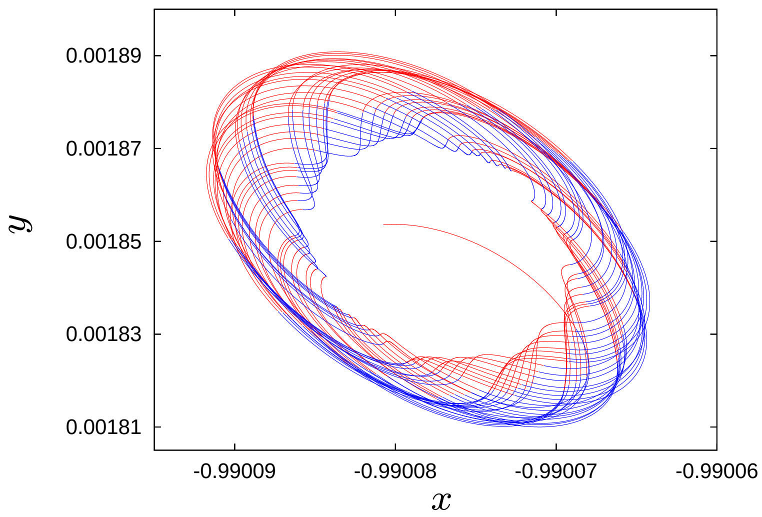
- Consider a spacecraft of mass 160 Kg and sail area 125×125 m^2 then the sail lightness number is $\beta = 0.15$.

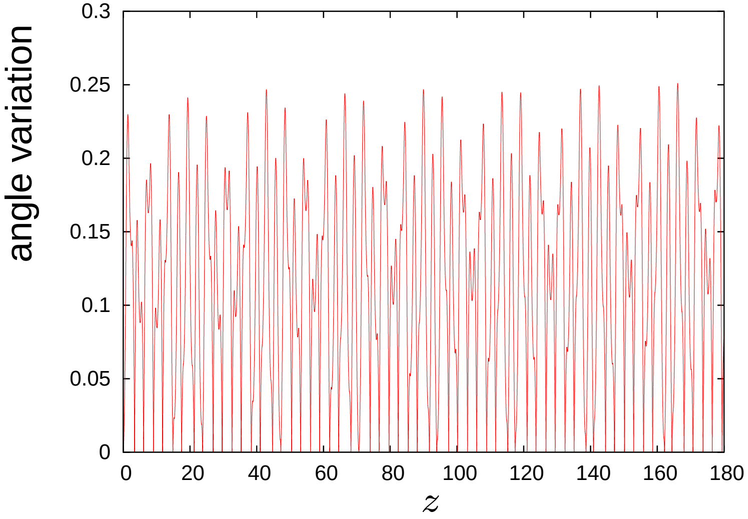
- We have considered two points on the $\{x, y\}$-plane at 10.12^o and 11.02^o from x-axis.

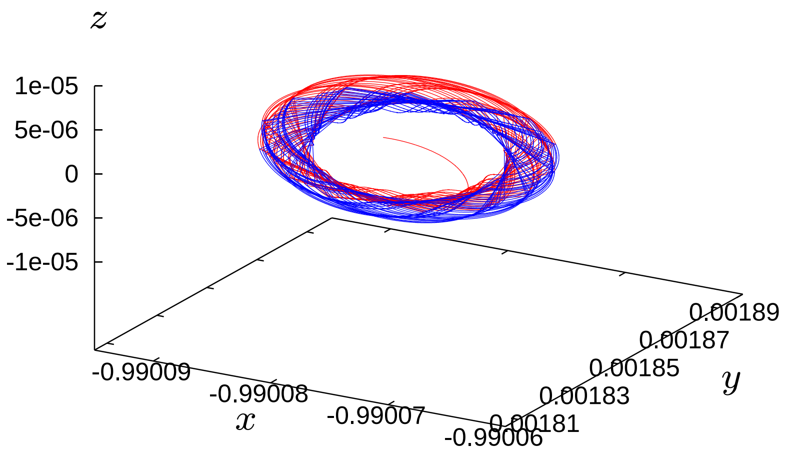
- Distance between the points 3.353893×10^4 Km (2.241939×10^{-4} AU).

- Manoeuvres every $30 - 25$ days.

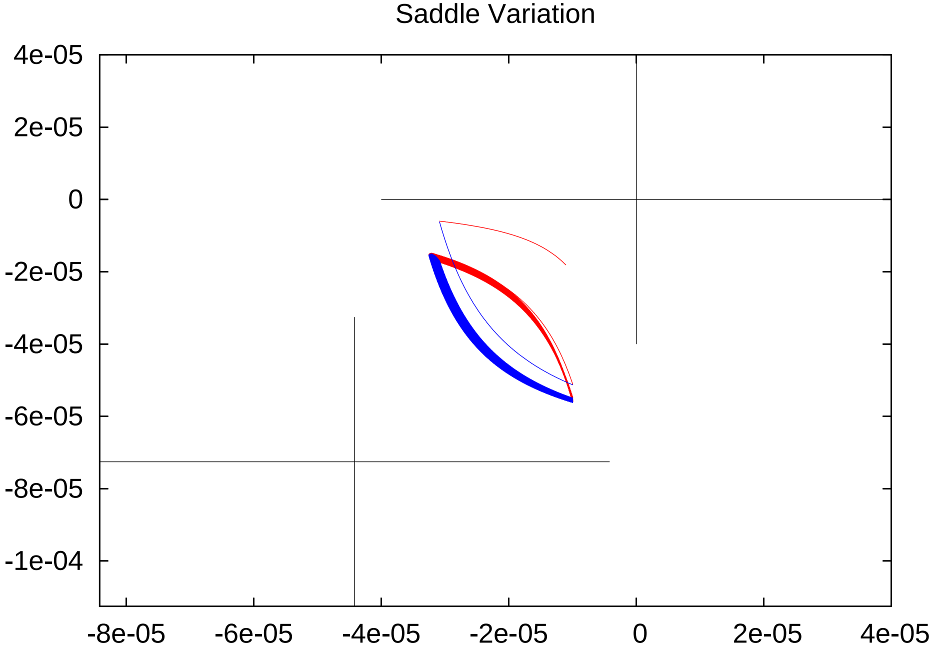
Results

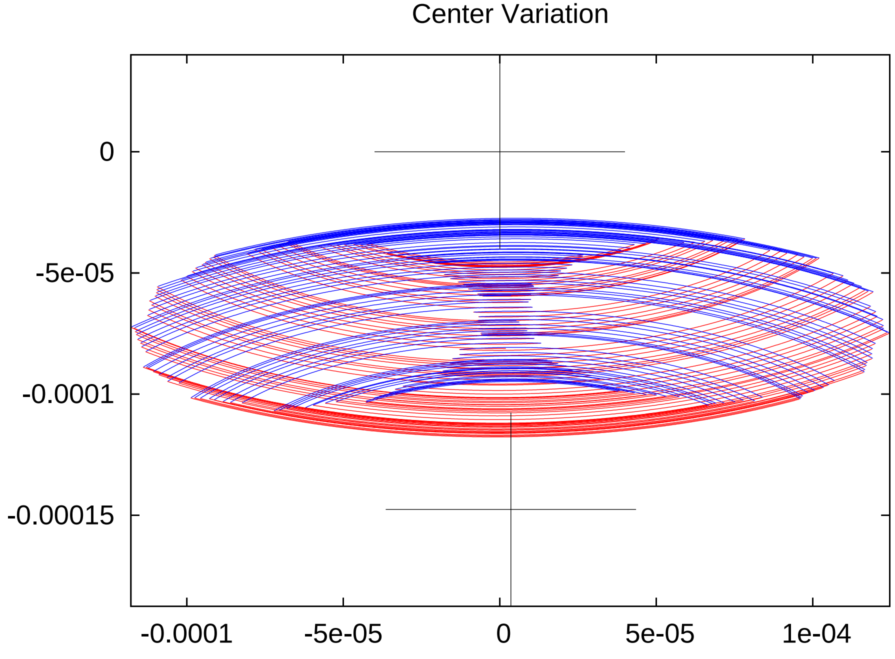

0.00189
0.00187
0.00185
0.00183
0.00181
y
-0.99009
-0.99008
-0.99007
-0.99006
x


angle variation
0.3
0.25
0.2
0.15
0.1
0.05
0
0
20
40
60
80
100
120
140
160
180
z


z
1e-05
5e-06
0
-5e-06
-1e-05
-0.99009
-0.99008
-0.99007
-0.99006
x
0.00189
0.00187
0.00185
0.00183
0.00181
y

Results


Saddle Variation
4e-05
2e-05
0
-2e-05
-4e-05
-6e-05
-8e-05
-1e-04
-8e-05
-6e-05
-4e-05
-2e-05
0
2e-05
4e-05


Center Variation
0
-5e-05
-0.0001
-0.00015
-0.0001
-5e-05
0
5e-05
1e-04

Conclusions

- We have used Dynamical Systems tools to control the Solar Sail near equilibrium points.
- The procedure is quite general.
- These techniques can be used to drift along the families of equilibrium points.
- They can also be used to control the motion around periodic orbits as for example halo orbits.