Workshop on Stability and Instability in Mechanical Systems: Applications and Numerical Tools

Barcelona, December 1-5 2008

Abstracts

Roberto Barrio: *Automatic differentiation, chaos indicators and dynamics*
In this talk we present the application of automatic differentiation techniques for the efficient numerical solution of ODEs and for the sensitivity equations up to any order via the extended Taylor series method. We show applications in several chaos indicators, which are applied to some paradigmatic dynamical systems: the Henon-Heiles Hamiltonian as example of open Hamiltonians and the Lorenz model as example of dissipative systems. The numerical results obtained from the indicators are complemented with other numerical or theoretical information, providing a more complete picture of the system.

Luis Benet: *Structure in narrow planetary rings: The scattering approach*
We address the emergence of the structural properties of narrow planetary rings within a general scattering framework. We consider the problem as Hamiltonian scattering of many independent massless particles in an effective potential, which involves some intrinsic rotation. We illustrate the consequences using as example a billiard system rotating on a Kepler orbit. We obtain rings that display sharp edges, non-zero eccentricity and variable width. In addition, they may show multiple components which are entangled in a similar way to the observed strands and braids, and may undergo processes which yield incomplete rings qualitatively similar to the observed clumps and arcs. Preliminary results using the restricted three-body problem will be briefly discussed.

Luca Biasco: *Low–order resonances in weakly dissipative spin–orbit models*
Second order differential equations with small nonlinearity and weak dissipation, such as the spin-orbit model of Celestial Mechanics, are considered. Explicit conditions for the coexistence of periodic orbits and estimates on the measure of the basins of attraction of stable periodic orbits are discussed.

Sergey Bolotin: *Second species solutions of the 3 body problem*
Abstract: We discuss second species solutions of Poincare for the plane 3 body problem with two of the masses small. Such solutions shadow chains of collision orbits of two uncoupled Kepler problems. Levi-Civita regularization replaces double collisions of small masses by a 2-dimensional normally hyperbolic symplectic invariant manifold. The problem is reduced to dynamics of a finite collection of symplectic maps. In certain cases this dynamics can be understood which gives a description of a large set of second species solutions.

Florentino Borondo: *Homoclinic dynamics and actions in quantum mechanics*
The pioneering work of Poincare and others at the turn of the twentieth century unveiled the possibility of chaotic motion in dynamical systems. Moreover, it demonstrated the importance of periodic orbits (PO), and its homoclinic and heteroclinic connections, in the hierarchical organization of the associated tangle.

In 1984, Heller published his seminal work on scar theory, in which the importance of POs was also demonstrated for quantum dynamics. Recently, and using a technique previously developed by us to construct scarred functions, we have demonstrated that the information concerning the associated homoclinic and heteroclinic motions is also contained in the quantum mechanics of the system. This extends Heller's work in the sense of considering the fate of the quantum probability not circulating along the main "scarring path" due to the PO, but pushed away, in the first instance by the "Lyapunov dynamics", and moving along homoclinic and heteroclinic circuits reinforcing the scarring power of the PO.

Moreover, when the quantization of some of these circuits is considered, the emergence of "homoclinic quantum numbers" and the importance of and associated topological phase, possibly connected with the Lazutkin invariant is observed.

Maciej Capiński: *Topological method for the detection of normally hyperbolic invariant manifolds*

We present a new topological proof of the existence of normally hyperbolic invariant manifolds for maps. The proof is conducted in the phase space of the system. In our approach we do not require that the map is a perturbation of some other map for which we already have an invariant manifold. We provide conditions which imply the existence of the manifold within an investigated region of the phase space. The required assumptions are formulated in a way which allows for rigorous computer assisted verification. We also present the construction of stable and unstable manifolds of the normally hyperbolic manifold, together with their foliations.

Fernando Casas: *Splitting methods in geometric numerical integration of differential equations*

In this talk we survey splitting and composition methods for the numerical integration of ordinary differential equations (ODEs). Splitting methods constitute an appropriate choice when the vector field associated with the ODE can be decomposed into several pieces and each of them is integrable. This class of integrators are explicit, simple to implement and preserve structural properties of the system. In consequence, they are specially useful in geometric numerical integration. In addition, the numerical solution obtained by splitting schemes can be seen as the exact solution to a perturbed system of ODEs possessing the same geometric properties as the original system. This backward error interpretation has direct implications for the qualitative behavior of the numerical solution as well as the error propagation along the time. Closely connected with splitting integrators are composition methods. We analyze the order conditions required by a method to achieve a given order and summarize the different families of schemes one can find in the literature. Finally, we illustrate the main features of splitting and composition methods on several numerical examples arising from applications.

Cristel Chandre: *Hamiltonian formulation of reduced Vlasov-Maxwell equations. Application to the Free Electron Laser*

We present a Hamiltonian formulation of the reduced Vlasov-Maxwell equations which is expressed in terms of the macroscopic fields. These macroscopic fields are themselves expressed in terms of the Lie-transform operators generated by some functional from the Poisson bracket of the exact Vlasov-Maxwell equations. We deduce the polarization and magnetization vectors from the Poisson bracket between this functional and the Maxwell-Vlasov fields.

Next, we further reduce the dynamical system in order to take into account the main approximations of a Free Electron Laser setting. From the original Maxwell-Vlasov equations, we derive a well known Hamiltonian model for the Free Electron Laser (Bonifacio's model) using a fully Hamiltonian framework.

Ariadna Farrés: *Nonlinear dynamics near equilibrium points for a Solar Sail*

We are interested in understanding the natural dynamics of a Solar Sail in the Earth - Sun system. We have considered the RTBP adding the radiation pressure to model the motion of the sail. The force due to the sail depends on three perimeters, two angles α and δ that define its orientation w.r.t the Sun-line direction, and β the sail's lightness number, which measures the sail's effectiveness.

In this work will fix $\alpha = 0$. Then, for each β, there are 1-d families of equilibrium points on the x,z-plane. For the values of β considered in the work all these equilibrium points have one pair of real eigenvalues ($\pm\lambda$) and two pair of complex eigenvalues ($\pm i\omega_1, \pm i\omega_2$). So around these equilibrium points we have families of unstable periodic orbits.

We will compute the centre manifold around these equilibrium points and the families of periodic orbits to have a complete understanding of the non-linear dynamics around these equilibrium points. We will see how these families evolve as we vary the sail orientation (δ).

This is joint work with A. Jorba.

Philip Holmes: *Neuromechanical models of animal locomotion*

I will describe mathematical models of legged locomotion that combine descriptions of neural pattern generator circuits with nonlinear muscle and body-limb dynamics in a single hybrid dynamical system (hybrid because vectorfields change at leg touch-down and lift-off events). Using such models we have shown that rapidly running insects can produce stable gaits and recover from substantial perturbations without extensive neural feedback. I will describe neural, classical-mechanical (piecewise-holonomic constraints), and mathematical aspects of the models.

This is joint work with Raghavendra Kukillaya and Joshua Proctor.

Charles Jaffé: *Transition states, stability, and correlated dynamics*

I will discuss recent work on relationship between correlated dynamics and the stability of transition states.

Àngel Jorba: *Parallel computation of invariant tori*

The numerical approximation of invariant tori of flows (or maps) is a computationally intensive task, specially when the tori is of dimension strictly larger than 2 (or 1 for maps). In this talk we will discuss several ways of taking advantage of the parallel capacities offered by a cluster of computers. We will use examples coming from problems of Astrodynamics, that require the computation of 3-D and 4-D invariant tori.

This is based on joint works with E. Castellà and E. Olmedo.

Ugo Locatelli: *On the effective stability in the neighbourhood of a KAM torus*

We reconsider the problem of the stability in the neighbourhood of a KAM torus into the light of the approach due to Morbidelli and Giorgilli (J. Stat. Phys., 1994). They proved that the diffusion time is superexponentially large with respect to the inverse of the distance from a fixed invariant torus. Here, we reformulate their result so to provide explicit estimates that can apply to physical problems. Moreover, we implement a *computer–assisted proof* for a model of the forced pendulum(s). As a byproduct of this approach, we can provide lower bounds on the relative measure of the KAM tori in the neighborhood of a fixed one. These results can actually be compared with those of some numerical experiments. This allows us to draw some conclusions about the optimality of our estimates.

The talk is based on a work in collaboration with A. Celletti and A. Giorgilli.

Jean-Pierre Marco: *A dynamical notion of complexity for integrable Hamiltonian systems*

Let us consider an integrable Hamiltonian H on a compact symplectic manifold of dimension $2n$, that is a system for which exists a first integral $F : M \to R^{2n}$ whose components are in involution and almost everywhere independent. Our problem is to analyze the dynamical complexity of the Hamiltonian flow generated by H. Under quite mild additional assumptions on the singular set of F the topological entropy of this flow vanishes. One is therefore led to introduce more refined tools. We will define two invariants which detect the polynomial growth rate of the complexity of orbits and are well-suited for the study of the complexity of integrable systems. This may be seen as a dynamical approach of the problem investigated by Fomenko and his school. We will show the influence of the singularities of F on the values of these invariants for non degenerate Hamiltonian systems on surfaces and on 4 dimensional manifolds.

Josep J. Masdemont: *Dynamical systems tools applied to transfer orbits in spacecraft mission design*

Invariant manifolds of the collinear libration point orbits in the restricted three body problem gives us the fundamental structure to understand transport phenomena from a geometric point of view.

In this presentation we give some background and analyze special techniques and tools with applications to practical problems in spacecraft mission design. In particular we will see how the invariant manifolds and their intersections (homoclinic and heteroclinic orbits) can play a key role in the design of different transfer problems.

Paul Milewski: *Stability of wavepacket solitary waves*

Wavepacket solitary waves resemble wavepackets –a high frequency carrier wave modulated slowly in amplitude– where both the envelope and the carrier travel at the same speed. Generically they bifurcate from linear waves when the dispersion relation (spectrum) has an extremum in the phase speed at finite wavenumber. They occur in free surface water waves in the capillary-gravity regime, but are difficult to observe physically due to viscous effects. Here we consider the stability and dynamics of these waves in the inviscid regime. We present a new reduced model equation (in the spirit of the Kadomtsev- Petviashvilli Equation) in 2+1 dimensions that governs these waves and show some asymptotic and numerical results on the stability of both plane and fully localized waves. We also present examples of collisions between solitary waves which suggest that the equation is not integrable. Lastly, we discuss the relation between this new model and standard envelope models such as the Nonlinear Schrodinger Equation.

Josep-Maria Mondelo: *Numerical Fourier analysis of quasi-periodic functions*

A procedure for the numerical computation of frequencies and amplitudes of quasi–periodic functions from equally–spaced samples will be presented. It is based on a collocation–like strategy in frequency domain, using the Discrete Fourier Transform. Comments will be made on the practical choice of parameters in order to obtain high precision, avoiding DFT–related phenomena (leakage, aliasing). An application will be given to the study of the dynamics in the (practical) stability zone around the triangular libration points of the planar, circular RTBP for the Sun-Jupiter mass ratio.

Joint work with G. Gómez and C. Simó.

Arturo Olvera: *Regularity properties of critical invariant circles of twist maps and their universality*
We accurately compute the golden and silver critical invariant circles of several area-preserving twist maps of the cylinder. We define some functions related to the invariant circle and to the dynamics of the map restricted to the circle. The global Holder regularities of these functions are low. We present several conjecture about the universality of the regularity properties of the critical circles and the related functions. Using a Fourier analysis method, we compute numerically the Holder regularities of these functions. Our computations show that these regularities are the same for the different maps studied. We discuss how our findings are related to some previous results: (a) to the constants giving the scaling behavior of the iterates on the critical invariant circle and (b) to some characteristics of the singular invariant measures connected with the distribution of iterates. Some of the functions studied have pointwise Holder regularity that has different values at different points. Our results give convincing numerical support to the fact that the points with different Holder exponents of these functions are interspersed in the same way for different maps, with is a strong indication that the underlying twist maps belong to the same universality class. In particular, the numerical results on the regularity of the so-called big conjugacies imply that the Holder spectra of the functions conjugating the dynamics on the critical invariant circle to a rigid rotation are the same. This, in turn, shows that the invariant measures on the critical circles have the same regularity spectra.

Hinke Osinga: *Invariant polygons and resonances in a friction oscillator*
We investigate a system with a sliding surface that has an unstable periodic orbit of focus type. This scenario is typical for a forced friction oscillator, which we use as the leading example. Sliding corresponds to sticking in this model. The instability of the periodic orbit is induced by the friction coefficient, which decreases with increased relative speed between the contacting surfaces. We derive a normal form return map near the onset of this instability and show that attracting invariant polygons arise in the system. We are able to construct a fractal-like bifurcation diagram that shows how the number of sides of the polygon varies as a function of the parameters. The polygons can be viewed as the analogon of an invariant torus for piecewise-smooth systems. Indeed, the dynamics on the polygons is phase-locked in large regions of the parameter space that form so-called Arnol'd tongues. However, not all is equivalent to smooth systems! The Arnol'd tongues look more like sausages and there is more to the polygons than the eye can see.
This is joint work with Robert Szalai.

Rytis Paskauskas: *Dynamical bottlenecks to intramolecular energy flow*
Vibrational energy flows unevenly in molecules, repeatedly going back and forth between trapping and roaming. We identify bottlenecks between diffusive and chaotic behavior, and describe generic mechanisms of these transitions, taking the carbonyl sulphide molecule OCS as a case study. The bottlenecks are found to be lower-dimensional tori; their bifurcations and unstable manifolds govern the transition mechanisms.

Jesús Peláez: *Dynamics and stability of tethered satellites at Lagrangian points*
The Lagrange equilibrium solutions of the Circular Restricted Three Body Problem (CRTBP) have turned out to be much more important than they seemed at a first sight. There have been many space applications in the past, and at present, which have used these libration points as essential elements of space missions (see [1]). They will likely play a much more important role

for future space exploration missions. Particularly attractive are the collinear points (L_1, L_2 and L_3) given their location and accessibility. Unfortunately all of them are unstable, which means a spacecraft to be kept at or orbiting around them will require correction manoeuvres typically to be performed at the expense of propellant mass. Colombo, one of the pioneer about the use of space tethers showed in [2] the feasibility of controlling the unstable nature of the collinear Lagrangian points exploiting a varying-length tether system. Such a concept was later studied more deeply by Farquhar [3, 4]. This control scheme, which exploits the capability of dumbbell systems of shifting the centre of gravity position with respect to the centre of mass in a controlled manner, allows keeping the position of an artificial satellite close to the Lagrangian points without using propellant. Misra et al. [5] later studied the problem with a different approach and included the possibility of using rotating tethered system of constant length to ease the stabilisation process. In the latter study the dynamics of a passive (constant-length) rotating dumbbell system were investigated without addressing possible control strategies. The need for further, more in-depth dynamic and control analysis of the system was highlighted.

In general, tether dynamics is complex (see [8]). Close to the collinear points the dynamics of the tether is influenced by many different factors. One of them is ν the reduced mass of the primary around which the tether is moving. In some cases this parameter is very small and the Hill approximation, successfully used in [7], allows a much more simple description of the dynamics which permits to gain an insight into the complex evolution of the tethered system. For these cases the Hill formulation directly provides an excellent approximation to the problem. Moreover, it gives significant clues for the analysis of the general case in which ν is of order unity; this way the approximation makes easier the indispensable numerical analysis associated with a detailed description of the dynamics.

We investigate the dynamics of a tethered system near the collinear libration points exploiting the benefits of the Hill approach and the variation of the tether length. We try a simple strategy that permits, using a feedback control law, to stabilize the system around equilibrium positions which are basically unstable. Rotating tethers, with constant or variable length, are also investigated. We focus the analysis on the effects of inert rotating tethers on different orbits and situations typical of the dynamics in the presence of a binary system.

More recently Pelaez and Scheeres [6, 7] proposed to place electrodynamic tethers for permanent power generation at points in the neighborhood of the Lagrangian points of the inner Jupiter moonlets (Metis, Adrastea, Amalthea and Thebe). The electrodynamic tether will be *deorbiting* the moonlet by using its gravitational attraction; in doing so it converts the mechanical energy of the moonlet into electrical energy that can be used onboard. As a consequence, a continuous power can be extracted from the orbital energy of the moonlet. In that case, current control was proposed as the sole mean to stabilize both the position and the attitude of a constant-length non-rotating electrodynamic tether system placed in the vicinity of the unstable Lagrangian points of the moonlets. The dynamical analysis of [7] shows that there exist equilibrium positions where the tether could be operated appropriately. Some of these equilibrium positions are **stable** and other **unstable**. At first sight, the operation of the probe in an stable equilibrium position would be preferable; however, the probe would be operated in an unstable equilibrium position if the corresponding control problem has a reliable solution.

Any mission to the Jovian world must face some hindering factors. The scientific payload is usually an small fraction of the whole S/C mass due to launcher limitations and the low dry/wet mass fraction characteristic of the chemical propellants. Moreover, solar panels become rapidly ineffective further from the Sun. The solar intensity at Jupiter, 5 AU distant from the Sun, is only one twenty-fifth of its value at Earth. As a consequence, energy is a *scarce commodity* in this kind of missions and the total energy which will be consumed by the spacecraft should be transported onboard. But the power source used, Radioisotope Thermoelectric Generators (RTGs), are relatively weak or require large masses which in the end strongly penalize the mission scientific payload. A mission to Io should be different of an Io Orbiter similar to the Europa Orbiter (EO) proposed by the NRC in 1999; the experience gained with the EO permits to consider as unrealistic an Io Orbiter for the next decade. The other option frequently considered, some repeated flyby missions to Io, appears clearly insufficient.

The main result of [6, 7] is to show that is possible to operate an electrodynamic tether at Metis to produce a larger level of electrical power than the one provided by the classical RTGs. Thus, with an Aluminum tape 40 km long, 10 mm wide and 0.1 mm thick, 4000 watts of power can be obtained in a continuous and sustained way (the mass of this tether would be, about 108 kg).

We extend the analysis of [6, 7] to the case of the Io moon. We analyze the pros and cons of this option. Since the orbital radius of Io (421,800 km) is larger than the radius of Metis (128,000 km) the Jovian magnetic field will be weaker and the performances of the bare tether as colector of the electrons of the environmental plasma will be smaller. However, the tether will be operated in the neighborhoods of the peak exhibited by the plasma density which is associated with the iogenic plasma source that is created near Io. This plasma density peak compensate a certain fraction of the drop in the useful power produced by the tether due to the drop of the Jovian magnetic field. The stability issue related to the tether tension is also addressed.

All the material has been taken from reference [9].

References

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Ettore Perozzi: *Space manifold dynamics: the pragmatic point of view*

The term "Space Manifold Dynamics (SMD)" has been recently proposed for encompassing the various applications of the dynamical systems approach to space mission design, ranging from the exploitation of halo orbits around the collinear lagrangian points to lunar transfers and satellite tour design. The connection of SMD to known dynamical behaviour of solar system bodies is also emerging, as witnessed by the role of the Tisserand invariant in allowing both the temporary satellite capture of comets by the giant planets and the ballistic capture of a spacecraft from the Moon. Within this framework the advantages and drawbacks of using SMD for exploration missions are reviewed with special emphasis on the "operational" aspects and on the "effective stability" issue.

Dolors Puigjaner: *Dynamics of particle trajectories in a Rayleigh-Bénard problem*

The determination and control of fluid mixing efficiency is an important issue in many engineering applications. Efficient mixing is usually related to turbulent regimes which are typically achieved by means of mechanical devices. However there are several industrial applications that require an efficient mixing in the absence of turbulence or high shear stresses developed close to the blades of impellers. In such cases, concepts and tools from dynamical systems theory can be applied to understand the mixing process.

The rich dynamics of fluid particle trajectories for the Rayleigh–Bénard problem inside a cube with perfectly conducting lateral walls has been investigated. The system of three differential equations of motion for each particle has been integrated explicitly in time. The velocity and temperature fields of the stationary flow solutions have been obtained by means of a parameter continuation procedure based on a Galerkin spectral method. Since the convective flows considered were steady the inert particle paths (stream traces) coincide with flow streamlines. The dynamics of the resulting fluid particle paths has been studied for five branches of stationary solutions and different values of the Rayleigh number within the range $10^4 \leq Ra \leq 1.5 \times 10^5$.

The stability properties and bifurcations of fixed points, including the ones at the boundaries, which play a key role in the global dynamics, have been analyzed. Main periodic orbits and their stability properties have also been determined. Some topological relations have been considered to check the results obtained. In addition the symmetries of the solutions have been exploited to obtain invariant closed surfaces where the velocity field is everywhere tangent to the surface. These invariant surfaces are important because they separate the domain into non-communicating regions, that is, into regions between which there is no transport.

Regions with regular motion, where nested tori and stable periodic orbits develop, have been identified in suitable Poincaré maps. The size and shape of regular regions have been numerically estimated by dividing the domain into 8×10^6 cells of equal size, integrating the trajectories of 49 fluid particles, and identifying the cells not visited by any fluid particle after a large integration time of 10^6.

The chaotic nature of the flows has been quantified by the maximal Lyapunov exponent and the metric entropy. Lyapunov exponents can be interpreted as the long time average of the specific rate of stretching of fluid elements. However, since the volume V_c occupied by the chaotic zone also plays a role the estimates of maximal Lyapunov exponent is not enough to have a quantitative measure of the mixing properties of the flows. Hence the metric entropy has also been used as an indicator of the mixing properties of the large chaotic zone.

This is a joint work with F. Giralt, J. Herrero and C. Simó

Clark Robinson: *Shadowing orbits for transition chains of invariant tori*

A topological method is used to prove the existence of diffusing orbits for a Hamiltonian system with a priori hyperbolicity that shadow a transition chains of invariant tori connected by either heteroclinic orbits or Birkhoff zones of instabilities.

Philippe Robutel: *A numerical study of the Trojan dynamics*

After a brief presentation of the dynamical influence of the resonances on asteroid belt and on the Kuiper belt, we shall explore the long term dynamics of the Jovian Trojans. The resonant structure of this problem will be described as well as some of its dynamical implications like diffusion, escapes or confinement.

Pau Roldán: *Arnold's mechanism of diffusion in the spatial circular Restricted Three Body Problem*

We show the existence of Arnold's mechanism of diffusion in the spatial circular Restricted Three Body Problem using a semi-numerical argument.

Specifically, we consider the center manifold associated to the equilibrium point L_1 in the Sun-Earth system. For small values h of the energy, the center manifold restricted to the energy level $H = h$ is a normally hyperbolic invariant manifold Λ. In the normal form approximation, Λ consists of a (1 parameter) family of invariant tori.

We find a transition chain of tori in Λ such that the unstable whisker of a torus intersects transversally the stable whisker of another neighboring torus. Moreover, we show how to construct an orbit that shadows the transition chain.

The argument is semi-numerical, consisting of a combination of geometrical methods (NHIM, scattering maps), topological methods (correctly aligned windows) and numerical methods (normal form, numerical integration of trajectories, numerical intersection of st/unst manifolds).

We emphasize that the argument is completely constructive, so we can construct different transition chains (thus shadowing orbits) that exhibit interesting dynamics. Therefore the results can be applied to e.g. space mission design.

We remark also that the methods apply to the more general setting of a normally hyperbolic manifold whose stable and unstable manifolds intersect transversally, so the argument could be adapted to some other models different from the spatial RTBP.

This is joint work with Amadeu Delshams and Marian Gidea.

Mercè Romero-Gómez: *The formation of spiral arms and rings in barred galaxies from the dynamical systems point of view*

The origin of the spiral arms in galaxies have been studied since the 60s. The most accepted theory is that spirals arms are density waves that propagate from the centre of the galaxy. This theory, however, has some drawbacks, like that the spiral arms damp when they encounter the principal resonances of the galaxy. Our approach is more recent and it has a dynamical origin. Rings in barred galaxies are believed to be linked with the outer resonance of the galaxy.

We observe that both spiral arms and rings emanated from the ends of the bar, where the unstable equilibrium points are located. These are of the type saddle $\times$ centre $\times$ centre and, therefore, they have a family of unstable planar and vertical periodic orbits and invariant tori around them. Our work reveals that the invariant manifolds associated to the planar Lyapunov orbits are responsible for the global dynamics of both spirals and rings. The difference between spirals and the different types of rings arises from the presence of homoclinic, heteroclinic and transit orbits. We have also performed some comparisons with observations and the results match well.

This is joint work with J.J. Masdemont and E. Athanassoula.

Shane Ross: *Dynamical boundaries in a variety of mechanical systems*

The identification of frontiers between qualitatively different kinds of behavior in a dynamical system is important in many applications. Increasingly, systems of interest are determined not by analytically defined model systems, but by data from experiments or large-scale simulations. This two-part talk will address: (1) systems known analytically from which phase space structures (separatrices) controlling transport and stability can be computed, e.g., in orbital mechanics and biomechanics; and (2) an approach for identifying separatrices in mechanical systems (e.g., stable motion vs. failure in postural balance) where the dynamics is sampled by experiment. Applications to other areas, such as ship capsize prediction and the role of atmospheric transport barriers in plant pathogen ecology, will also be discussed.

Eli Shlizerman: *Parabolic resonance: A route to Hamiltonian spatio-temporal chaos*

We show that initial data near an unperturbed *stable* plane wave can evolve into a regime of spatio-temporal chaos in the slightly forced conservative periodic one-dimensional nonlinear Schrödinger equation. Statistical measures are employed to demonstrate that this spatio-temporal chaos is intermittent: there are windows in time for which the solution gains spatial coherence. The parameters and initial profiles that lead to such intermittency are predicted by utilizing a novel geometrical description of the integrable unforced equation.

This is a joint work with V. Rom-Kedar.

Carles Simó: *Efficient numerical implementation of integrability criteria based on high order variational equations*

Non-integrability criteria of Hamiltonian systems, based on differential Galois theory and requiring the use of higher order variational equations, can be difficult to check analytically, due to different reasons. Efficient numerical techniques to check these criteria are introduced for quick checks, applicable to arbitrary systems. Several examples will be discussed.

Arturo Vieiro: *Quantitative global phase space analysis of APM*

Our goal will be discuss local/global properties that allow to clarify phase space structure of nearly integrable APM. As it is well-known, the main characteristic of this structure is the coexistence of regular and chaotic behaviour in the phase space.

In the first part of the talk we will briefly review the main ideas of normal form theory and KAM theory concerning the existence of a stable domain surrounding elliptic points. From this analytical approach it can be obtained local information of the structure of resonances and, in particular, on the inner and outer splitting of a resonant island. Moreover, it will be stated that the inner and outer splittings are generically different given an analytical description in the case the resonant island emanating from an elliptic fixed point. Some comments on the special properties of strong resonances will be done.

Then, we will move to the second part of the talk where we will focus on describing dynamics within chaotic zones by terms of return map models. Concerning dynamics in a neighbourhood of the separatrices of a resonant island we will show how to obtain quantitative data by terms of the "double" (figure "eight") separatrix map model and we will describe the role played by the inner and outer splittings of resonant islands in the description of the dynamics. The case of strong resonances will deserve some peculiarities on the global properties that will be analysed.

On the other hand, overlapping of resonances give rise to large chaotic domains, the so-called Birkhoff zones of instability. To analyse dynamics in these zones we propose the study of the biseparatrix return map model. Several examples will illustrate the different scenarios.

Along the presentation we will stress the fact that in applications it becomes relevant not only a topological description of phase space but a quantitative one. In particular the Hénon map will be systematically used to check the results. This global quantitative description needs of analytical local tools together with global numerical approach.

Joint work with C. Simó.

Piotr Zgliczynski: *Resonance transitions for Oterma comet in the Sun-Jupiter system*

I will discuss some results related to resonance transition in the restricted three body problem relevant for explaining the motion of some comets in the Sun-Jupiter system. The results are the existence of symbolic dynamics in the circular problem their symmetry propeties and work in progress regarding the survival of these features in the elliptic problem.

The results are computer assisted. The method combines topological arguments and the rigorous numerics.

The talk will be based on the joint works with Daniel Wilczak and Maciej Capiński.