Solar Sailing in the Earth - Sun System

Séminaire Temps & Espace

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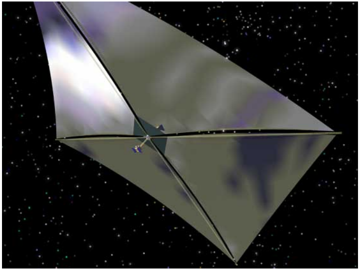
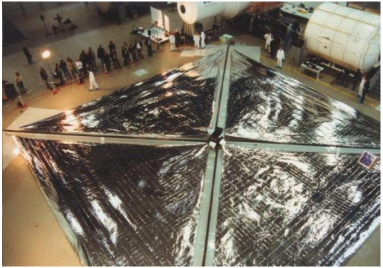
Background on Solar Sails

What is a Solar Sail ?

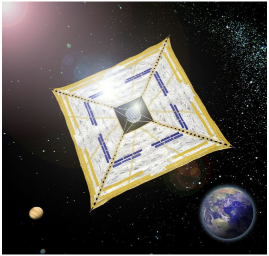
- Solar Sails a proposed form of propulsion system that takes advantage of the Solar radiation pressure to propel a spacecraft.
- The impact of the photons emitted by the Sun on the surface of the sail and its further reflection produce momentum on it.
- Solar Sails open a wide new range of possible missions that are not accessible by a traditional spacecraft.

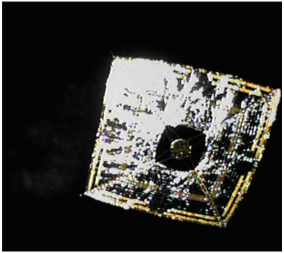




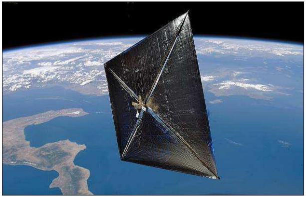
There have recently been two successful deployments of solar sails in space.

- IKAROS: in June 2010, JAXA managed to deploy the first solar sail in space.



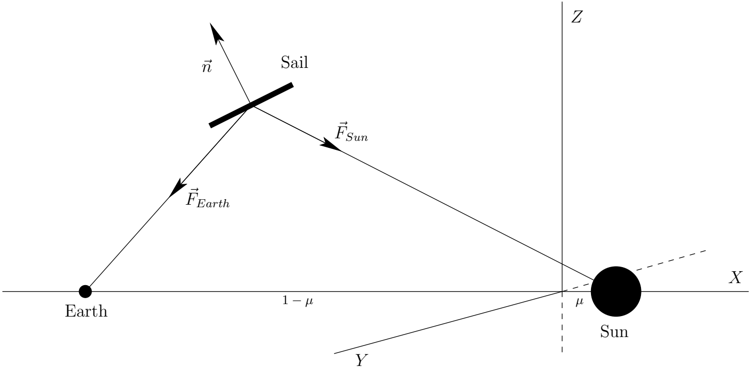


- NanoSail-D2: in January 2011, NASA deployed the first solar sail that would orbit around the Earth.



The Dynamical Model

We use the Restricted Three Body Problem (RTBP) taking the Sun and Earth as primaries and including the solar radiation pressure due to the solar sail.



The Solar Sail

We consider the solar sail to be flat and perfectly reflecting. Hence, the force due to the solar radiation pressure is in the normal direction to the surface of the sail.

The force due to the sail is defined by the *sail's orientation* and the *sail's lightness number*.

- The *sail's orientation* is given by the normal vector to the surface of the sail, $\vec{n}$. It is parametrised by two angles, α and δ.
- The *sail's lightness number* is given in terms of the dimensionless parameter β. It measures the effectiveness of the sail.

Hence, the force is given by:

$$\vec{F}_{sail} = \beta \frac{m_s}{r_{ps}^2} \langle \vec{r}_s, \vec{n} \rangle^2 \vec{n}.$$

Equations of Motion

The equations of motion are:

$$\begin{aligned}
\ddot{x} &= 2\dot{y} + x - (1-\mu)\frac{x-\mu}{r_{ps}^3} - \mu\frac{x+1-\mu}{r_{pe}^3} + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_x,\\
\ddot{y} &= -2\dot{x} + y - \left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right) y + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_y,\\
\ddot{z} &= -\left(\frac{1-\mu}{r_{ps}^3} + \frac{\mu}{r_{pe}^3}\right) z + \beta\frac{1-\mu}{r_{ps}^2}\langle\vec{r}_s,\vec{n}\rangle^2 n_z,
\end{aligned}$$

where $\vec{n} = (n_x, n_y, n_z)$ is the normal direction to the surface of the sail with,

$$\begin{aligned}
n_x &= \cos(\phi(x,y)+\alpha)\cos(\psi(x,y,z)+\delta),\\
n_y &= \sin(\phi(x,y,z)+\alpha)\cos(\psi(x,y,z)+\delta),\\
n_z &= \sin(\psi(x,y,z)+\delta),
\end{aligned}$$

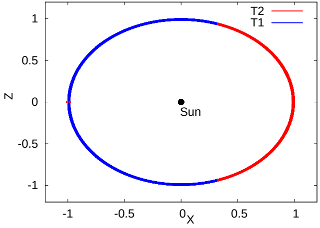
and $\vec{r}_s = (x-\mu, y, z)/r_{ps}$ is the Sun - sail direction.

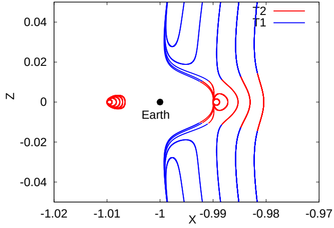
Equilibrium Points (I)

- The RTBP has 5 equilibrium points (L_i). For small β, these 5 points are replaced by 5 continuous families of equilibria, parametrised by α and δ.
- For a fixed small value of β, we have 5 disconnected family of equilibria around the classical L_i.
- For a fixed and larger β, these families merge into each other. We end up having two disconnected surfaces, S_1 and S_2. Where S_1 is like a sphere and S_2 is like a torus around the Sun.
- All these families can be computed numerically by means of a continuation method.

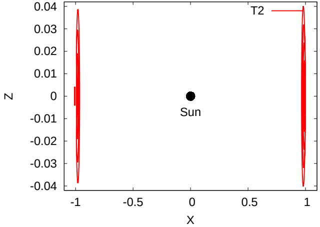
Equilibrium Points (II)

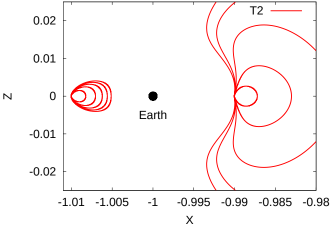
Equilibrium points in the XY plane





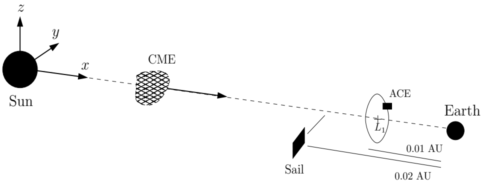
Equilibrium points in the XZ plane



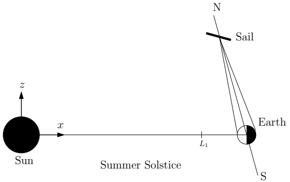


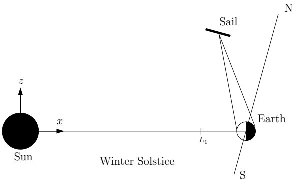
Interesting Missions Applications

Observations of the Sun provide information of the geomagnetic storms, as in the Geostorm Warning Mission.



Observations of the Earth's poles, as in the Polar Observer.





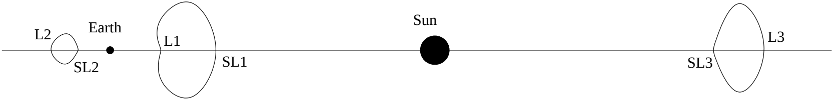
Periodic Motion Around Equilibria

We must add a constrain on the sail orientation to find bounded motion. One can see that when $\alpha = 0$ and $\delta \in [-\pi/2, \pi/2]$ (i.e. only move the sail vertically w.r.t. the Sun - sail line):

- The system is time reversible $\forall\delta$ by $R : (x, y, z, \dot{x}, \dot{y}, \dot{z}, t) \to (x, -y, z, -\dot{x}, \dot{y}, -\dot{z}, -t)$ and Hamiltonian only for $\delta = 0, \pm\pi/2$.
- There are 5 disconnected families of equilibrium points parametrised by δ, we call them $FL_{1,\ldots,5}$ (each one related to one of the Lagrangian points $L_{1,\ldots,5}$).
- Three of these families ($FL_{1,2,3}$) lie on the $Y = 0$ plane, and the linear behaviour around them is of the type saddle$\times$centre$\times$centre.
- The other two families ($FL_{4,5}$) are close to $L_{4,5}$, and the linear behaviour around them is of the type sink$\times$sink$\times$source or sink$\times$source$\times$source.

We focus on ...

- We focus on the motion around the equilibrium on the FL_1 family close to SL_1 (they correspond to $\alpha = 0$ and $\delta \approx 0$).
- We fix $\beta = 0.051689$ (loading parameter $\sigma \approx 30g/m^2$).
- We consider the sail orientation to be fixed along time.

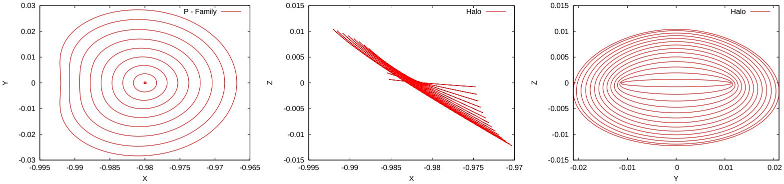


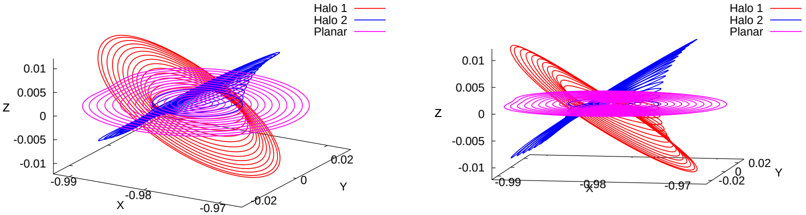
(Schematic representation of the equilibrium points on $Y = 0$)

Let us see the periodic motion around these points for a fixed sail orientation and show how it varies when we change, slightly, the sail orientation.

$\mathcal{P}$-Family of Periodic Orbits

Periodic Orbits for $\delta = 0$.

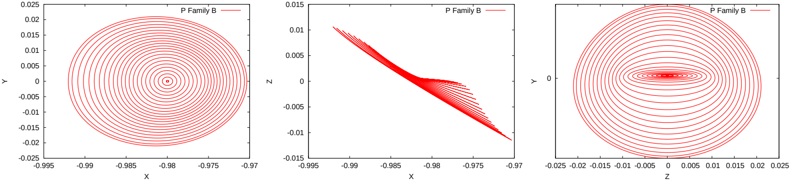




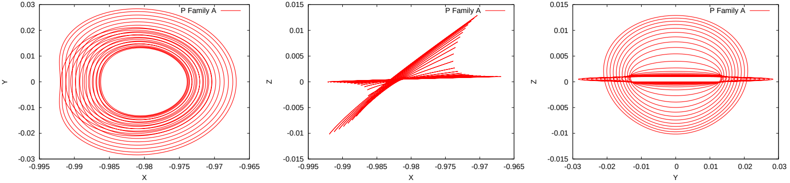
$\mathcal{P}$-Family of Periodic Orbits

Periodic Orbits for $\delta = 0.01$.

Main family of periodic orbits for $\delta = 0.01$

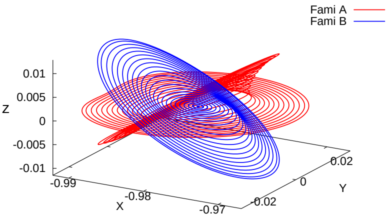


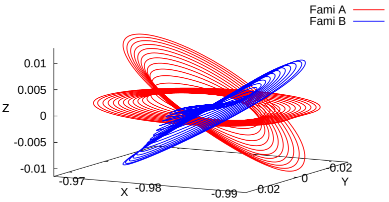
Secondary family of periodic orbits for $\delta = 0.01$

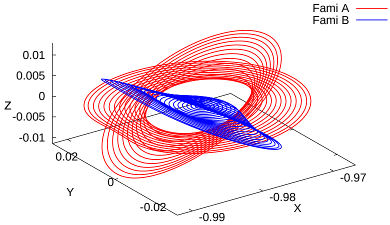


$\mathcal{P}$-Family of Periodic Orbits

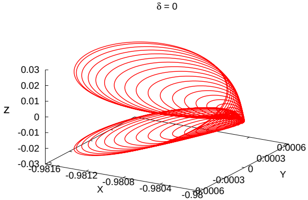
Periodic Orbits for $\delta = 0.01$.

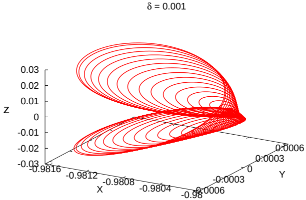


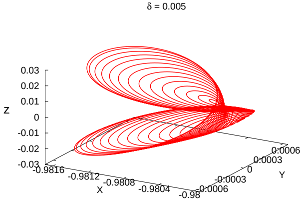


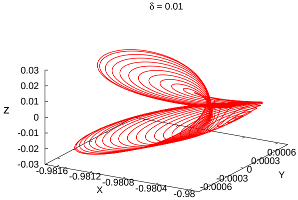


$\mathcal{V}$ - Family of Periodic Orbits









AIM of this TALK

One of the main goals of our work was to understand the geometry of the phase space and how it varies when the sail orientation is changed. Then use this information to derive strategies to control the trajectory of a Solar Sail.

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We will:

1. describe the dynamics of a solar sail around an equilibrium point (for a fixed sail orientation) and show the effects of variations on the sail orientation on the sail trajectory.

2. show how to use this information to derive a station keeping strategy around an equilibrium point, or surf along the families of equilibria.

Station Keeping Strategies Around Equilibria

Station Keeping for a Solar Sail

We want to design station keeping strategy to maintain the trajectory of a solar sail close to an unstable equilibrium point.

Instead of using *Control Theory Algorithms*, we want to use *Dynamical System Tools* to find a station keeping algorithm for a Solar Sail.

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Instead of using *Control Theory Algorithms*, we want to use *Dynamical System Tools* to find a station keeping algorithm for a Solar Sail.

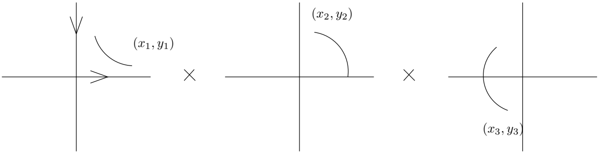
The main ideas are ...

- To focus on the linear dynamics around an equilibrium point and study how this one varies when the sail orientation is changed.
- To change the sail orientation (i.e. the phase space) to make the system act in our favour: keep the trajectory close to a given equilibrium point.

Station Keeping for a Solar Sail

We focus on the two previous missions, where the equilibrium points are unstable with two real eigenvalues, $\lambda_1 > 0, \lambda_2 < 0$, and two pair of complex eigenvalues, $\nu_{1,2} \pm \mathrm{i}\omega_{1,2}$, with $|\nu_{1,2}| << |\lambda_{1,2}|$.

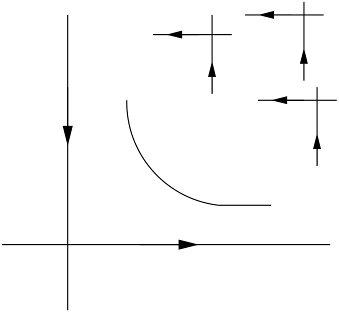
- To start we can consider that the dynamics close the equilibrium point is of the type saddle $\times$ centre $\times$ centre.
- From now on we describe the trajectory of the sail in three reference planes defined by each of the eigendirections.



- For small variations of the sail orientation, the equilibrium point, eigenvalues and eigendirections have a small variation. We will describe the effects of the changes on the sail orientation on each of these three reference planes.

Schematic Idea of the Station Keeping Strategy (I)

In the saddle projection of the trajectory:

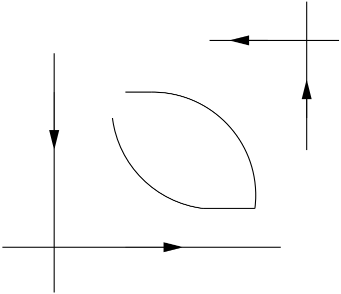


- When we are close to the equilibrium point, p_0, the trajectory escapes along the unstable direction.
- When we change the sail orientation the position of the equilibrium point is shifted and its eigendirections vary slightly.

Schematic Idea of the Station Keeping Strategy (II)

In the saddle projection of the trajectory:

- Now the trajectory will escape along the new unstable direction.
- We want to find a new sail orientation (α, δ) so that the trajectory will come close to the stable direction of p_0.



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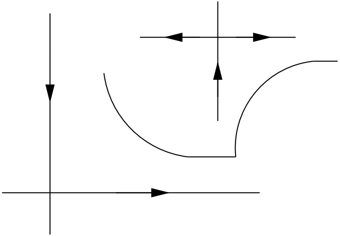

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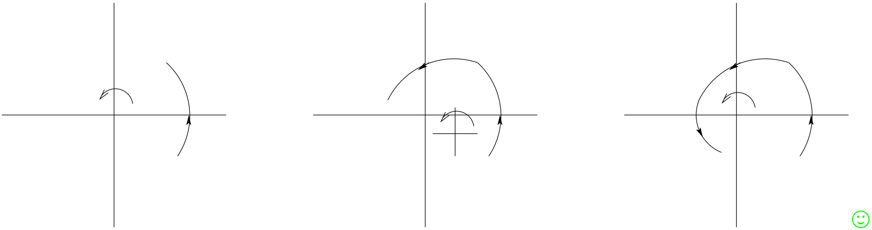
Schematic Idea of the Station Keeping Strategy (III)

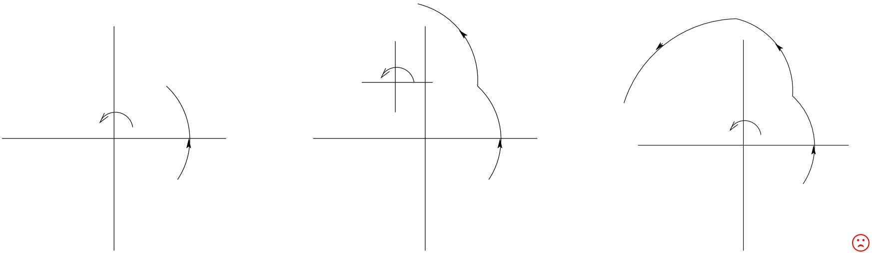
In the centre projection of the trajectory:



A sequence of changes on the sail orientation implies a sequence of rotations around different equilibrium points on the centre projection, which can result of an unbounded growth.

Schematic Idea of the Station Keeping Strategy (III)

In the centre projection of the trajectory:



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Schematic Idea of the Station Keeping Strategy (IV)

How can we choose the position of the new equilibrium point so that projection of the trajectory on the saddle and centre projections are bounded ?

Schematic Idea of the Station Keeping Strategy (IV)

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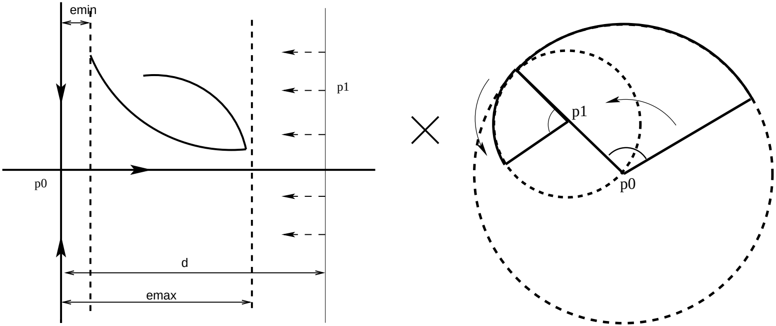


The constants ε_{min}, ε_{max} and d will depend on the mission requirements and the dynamics around the equilibrium point.

Observations

- We do not know explicitly the position of the equilibrium points $p(\alpha, \delta)$. But we can compute the linear approximation of this function

$$p(\alpha, \delta) = p(\alpha_0, \delta_0) + Dp(\alpha_0, \delta_0) \cdot (\alpha - \alpha_0, \delta - \delta_0)^T.$$

- There are some restrictions of the position of the new equilibria when we change α and δ. We have 2 unknowns and at least 6 conditions that must be satisfied.

- We will change the sail orientation so that the position of the new fixed point is as close as possible to the desired new equilibrium point and in the correct side in the saddle projection.

- To decide the new sail orientation we will assume that the eigenvalues and eigendirections do not vary when the sail orientation is changed.

- All the simulations have been done using the full set of equations, we only use the linear dynamics to decide the change on the sail orientation.

Scheme of the Station Keeping Algorithm

We look at the sails trajectory in the reference system $\{x_0; \vec{v}_1, \vec{v}_2, \vec{v}_3, \vec{v}_4, \vec{v}_5, \vec{v}_6\}$, so $z(t) = x_0 + \Sigma_i s_i(t)\vec{v}_i$. And we define the parameters d, ε_{max} and ε_{min}.

During the station keeping algorithm:

1. for $\alpha = \alpha_0, \delta = \delta_0$: if $|s_1(t)| \geq \varepsilon_{max}$ $\Rightarrow$ choose appropriate new sail orientation $\alpha = \alpha_1, \delta = \delta_1$.
2. for $\alpha = \alpha_1, \delta = \delta_1$: if $|s_1(t)| \leq \varepsilon_{min}$ $\Rightarrow$ restore the sail orientation: $\alpha = \alpha_0, \delta = \delta_0$.
3. Go Back to 1.

Results

We have applied this station keeping strategy to different mission scenarios. We show the results for the Geostorm Warning Mission.

For each mission:

- We have done a Monte Carlo simulation taking a 1000 random initial conditions.
- For each simulation we have applied the station keeping strategy for 30 years.
- We have tested the robustness of our strategy including random errors on the position and velocity determination, as well as on the orientation of the sail at each manoeuvre.

Results for the Geostorm

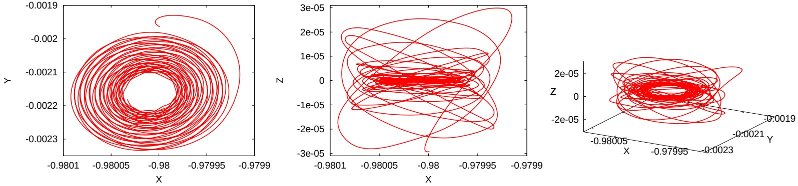
We have taken $\beta = 0.051689$ (i.e. a satellite of $130kg$ mass with a $67m \times 67m$ square sail).

	% Success	Max. Time	Min. Time	Ang. Vari.
No Error	100 %	45.87 days	24.13 days	1.43°
Error Pos.	100 %	45.85 days	24.13 days	1.43°
Error Pos. & Orient. *	100 %	53.90 days	21.59 days	1.42°
Error Pos. & Orient. †	97 %	216.47 days	15.54 days	1.67°

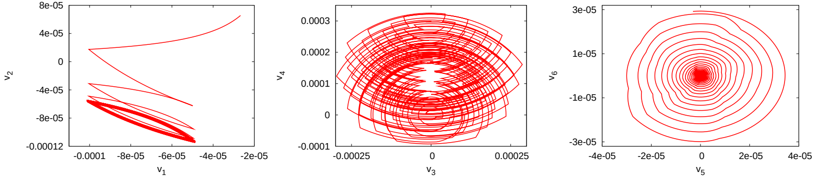
Statistics for the Geostorm mission taking 1000 simulations. Considering errors on the sail orientation of order 0.5° (*) and 2.2° (†).

Results for the Geostorm (No Errors in Manoeuvres)

XY and XZ and XYZ Projections



Saddle × Centre × Centre Projections



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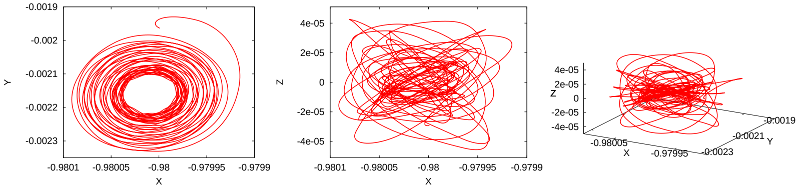

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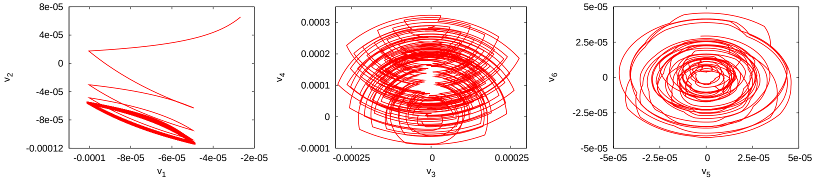
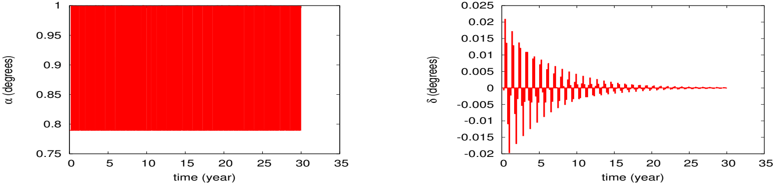
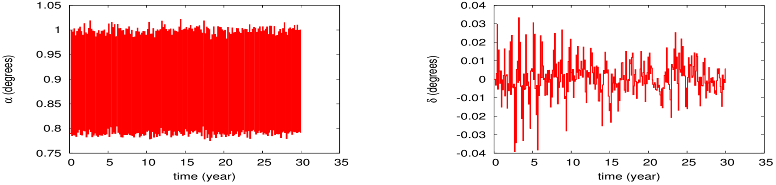


Results for the Geostorm

Variation of the sail orientation (No Errors in Manoeuvres)



Variation of the sail orientation (Errors in Manoeuvres)



Results

- We manage to maintain the trajectory close to the equilibrium point for 30 years.
- The most significant errors are the ones due to the sail orientation.
- The parameters ε_{min}, ε_{max} and d will vary depending on the local dynamics and the mission specifications.
- This station keeping strategy does not require previous planning as the decisions taken by the sail only depend on its position in the phase space.

[1] A. Farrés and À. Jorba, "A dynamical System Approach for the Station Keeping of a Solar Sail.", *Journal of Astronautical Science.* Volume 63, No. 2, April-July 2008, Pages 199-230.

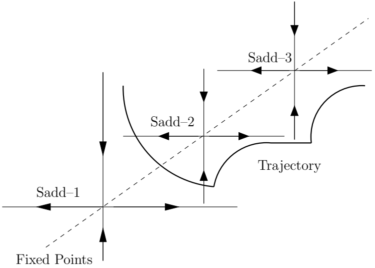
Surfing Along the Family of Equilibria

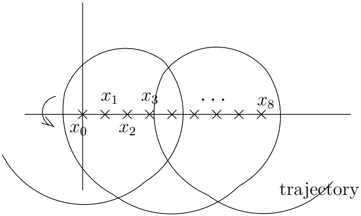
Surfing along the family of equilibria

- Using the same ideas, we can derive strategies to drift along the families of equilibria in a controlled way (use the invariant manifolds to move in the phase space).
- Lets assume that we are close to an equilibrium point p_0 and we want to reach the vicinity of another equilibrium point p_f. Once we reach p_f we want to be able to remain there for a long time.
- We want to find a sequence of changes on the sail orientation (α_i, δ_i) (i.e. a sequence of fixed points p_i) so that the sequence of stable/unstable directions of p_i guide the probe to the final point.
- Note that we also need to take into account the projection of the trajectory on the centre component.

Surfing along the family of equilibria

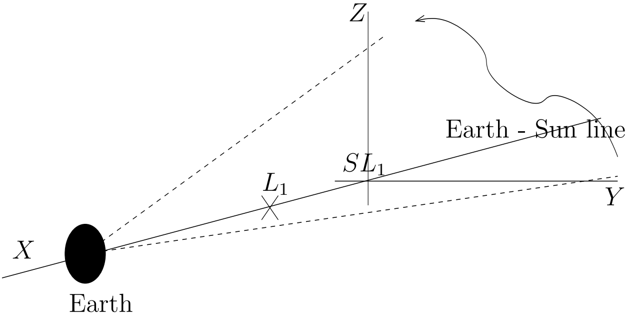
Scheme on the idea to surf along the family of equilibria.





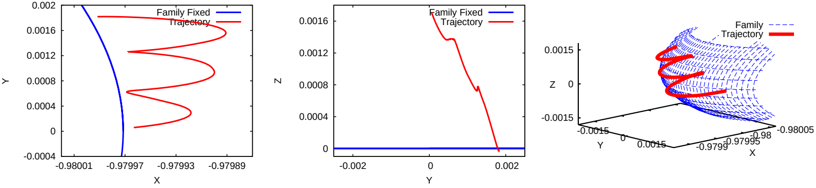
Proposed Missions / Toy Missions

We propose a toy mission to surf along the family of equilibria. We take the Geostorm mission scenario and we want to change the position of the solar sail.

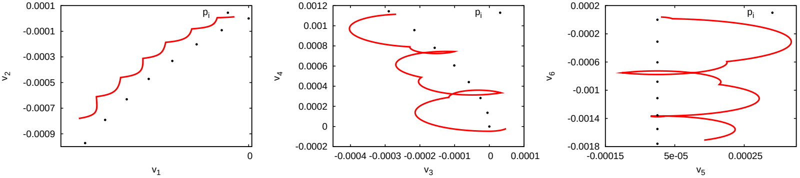


Surfing to gain vertical displacement

XY and XZ and XYZ Projections

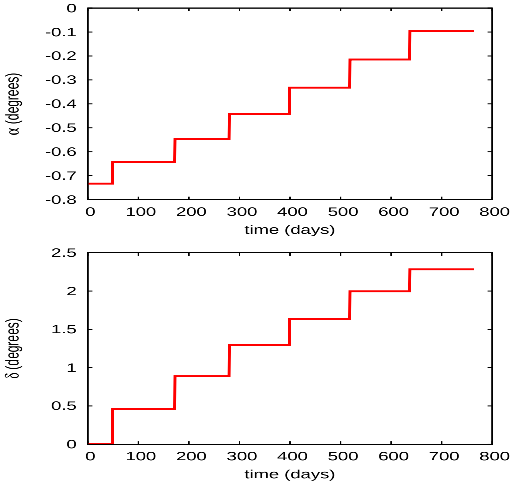


Saddle × Centre × Centre Projections



Surfing to gain vertical displacement

Variation of the sail orientation along time.



Towards a More Realistic Model

Including more realism to the dynamical model

There are several ways to include more realism to the dynamical model. For example,

- taking a more realistic model for the Solar Sail by including the force produced by the absorption of the photons, the reflectivity properties of the sail material, ...
- taking a more realistic model for the gravitational perturbations by including the eccentricity in the Earth - Sun system. Or the gravitational attraction of other bodies, i.e. the Moon, Jupiter, ...

We have started by considering the eccentricity in the Earth - Sun system and studied the robustness of our strategies. So we take the Elliptic Restricted Three Body Problem with a Solar sail as a model.

In the ERTBP + Solar Sail

The fixed points that existed in the RTBP + Solar sail no longer exist in this model. They have been replaced by periodic orbits of same period as the Earth's orbit around the Sun.

We can apply the same ideas to remain close to one of these periodic orbits and fulfil the mission requirements of the Geostorm mission or the Polar Observer.

Notice that:

- for each sail orientation (α, δ) we have an unstable periodic orbit replacing the fixed point.
- taking the Floquet modes we have a periodic reference system that will give us a good description of the local dynamics around these periodic orbits.
- we can apply the same ideas as before considering this appropriate reference system.

Station Keeping Strategies

The solution, $A(t)$, of the first order variational flow, $\dot{A} = DF(y(t))A$, gives us the information of the linear dynamics around a periodic orbit.

In a suitable basis the monodromy matrix $(A(\tau))$ associated to one of these periodic orbits can be written in the form,

$$B = \left(\begin{array}{cc|cc|cc} \lambda_1 & & & & & \\ & \lambda_2 & & & & \\ \hline & & \Delta_1 \cos\Gamma_1 & -\Delta_1 \sin\Gamma_1 & & \\ & & \Delta_1 \sin\Gamma_1 & \Delta_1 \cos\Gamma_1 & & \\ \hline & & & & \Delta_2 \cos\Gamma_2 & -\Delta_2 \sin\Gamma_2 \\ & & & & \Delta_2 \sin\Gamma_2 & \Delta_2 \cos\Gamma_2 \end{array} \right).$$

The functions $e_i(\tau) = A(\tau)e_i(0)$, $i = 1, \ldots, 6$, give an idea of the variation of the phase space properties in a small neighbourhood of the periodic orbit.

Station Keeping Strategies

We take the Floquet modes $\bar{e}_i(\tau)$ $i = 1, \ldots, 6$, six periodic functions that can easily be recovered by $e_i(\tau)$.

$$
\begin{aligned}
\bar{e}_1(\tau) &= e_1(\tau) \exp\left(-\tfrac{\tau}{T} \ln \lambda_1\right), \\
\bar{e}_2(\tau) &= e_2(\tau) \exp\left(-\tfrac{\tau}{T} \ln \lambda_2\right), \\
\bar{e}_3(\tau) &= \left[\cos\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_3(\tau) - \sin\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_4(\tau)\right] \exp(-\tfrac{\tau}{T} \ln \Delta_1), \\
\bar{e}_4(\tau) &= \left[\sin\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_3(\tau) + \cos\left(-\Gamma_1 \tfrac{\tau}{T}\right) e_4(\tau)\right] \exp(-\tfrac{\tau}{T} \ln \Delta_1), \\
\bar{e}_5(\tau) &= \left[\cos\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_5(\tau) - \sin\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_6(\tau)\right] \exp(-\tfrac{\tau}{T} \ln \Delta_2), \\
\bar{e}_6(\tau) &= \left[\sin\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_5(\tau) + \cos\left(-\Gamma_2 \tfrac{\tau}{T}\right) e_6(\tau)\right] \exp(-\tfrac{\tau}{T} \ln \Delta_2).
\end{aligned}
$$

These functions can be computed numerically. As they are periodic they can be spanned as Fourier series and easily stored by their Fourier coefficients.

Results

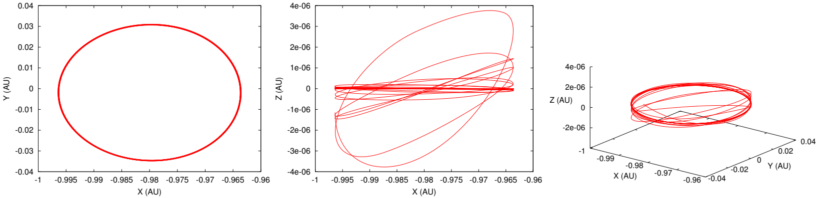
We have applied this strategies to the Geostorm Warning Mission scenario in the ERTBP for $e = 0.0167$. We have taken $\beta = 0.051689$ (i.e. a satellite of $130kg$ mass with a $67m \times 67m$ square sail).

	%	Δt	$\Delta\alpha$	$\Delta\delta$
No Error	100%	69.75 - 151.03 days	0.058^o - 0.0619^o	0^o - 0.003^o
Error Type 1	100%	69.77 - 150.9 days	0.058^o - 0.0619^o	0^o - 0.003^o
Error Type 2a	100%	67.28 - 166.9 days	0.057^o - 0.063^o	0^o - 0.004^o
Error Type 2b	100%	58.05 - 349.7 days	0.049^o - 0.069^o	0.00009^o - 0.018^o
Error Type 2c	47.8%	51.96 - 367.5 days	0.04^o - 0.075^o	0.0004^o - 0.02^o

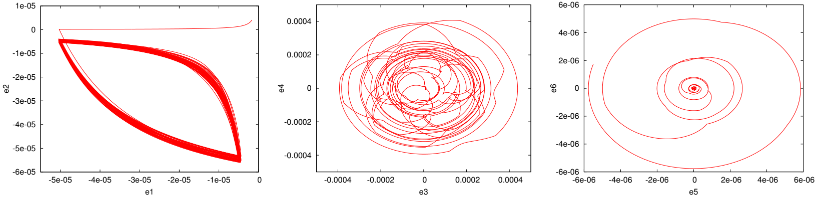
Statistics for 1000 simulations of the GeoStorm mission. Type 1 errors only consider errors on the position and velocity determination and Type 2 errors also consider errors on the sail orientation, for different maximum bounds for these errors. Type 2a is $0.057^o = 0.001rad$, *Type 2b is* $0.286^o = 0.005rad$ *and Type 2c is* $0.57^o = 0.01rad$.

Results for the Geostorm (No Errors in Manoeuvres)

XY and XZ and XYZ Projections

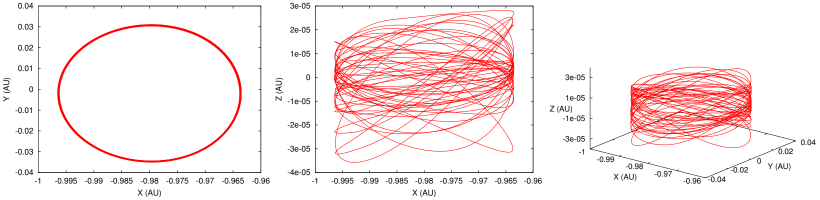


Saddle × Centre × Centre Projections

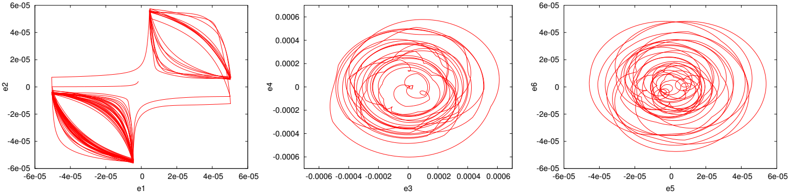


Results for the Geostorm (Errors in Manoeuvres)

XY and XZ and XYZ Projections

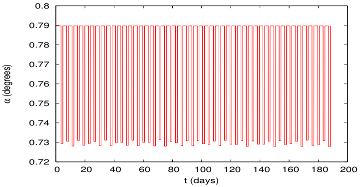


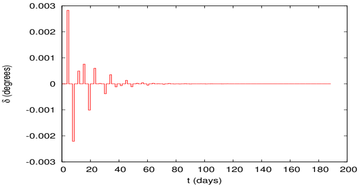
Saddle × Centre × Centre Projections



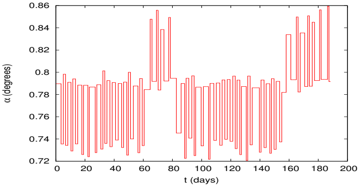
Results for the Geostorm

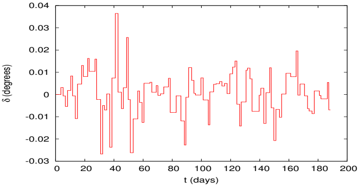
Variation of the sail orientation (No Errors in Manoeuvres)





Variation of the sail orientation (Errors in Manoeuvres)





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[1] Farrés, A. & Jorba, À.: *Dynamical system approach for the station keeping of a solar sail.* The Journal of Astronautical Science, 58:2, pp. 199-230, 2008.

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[3] Farrés, A.: *Contribution to the Dynamics of a Solar Sail in the Earth-Sun System* PHD Thesis, University of Barcelona, October 2009.

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A different approach for the station keeping

For all these strategies we have taken into account the information of the variation of the “fixed points” with respect to the sail orientation.

We are now working on a different approach to the problem, that is to use the variational equations of the flow.

The computation of variational equations (up to suitable order) w.r.t. α and δ gives explicit expressions for the effect of different orientations (close to the reference values $\alpha = \alpha_0, \delta = \delta_0$) trajectory.

$$\phi_t(y_0, \alpha_0 + h_a, \delta_0 + h_d) = \phi_t(y_0, \alpha_0, \delta_0) + \frac{\partial \phi}{\partial \alpha}(y_0, \alpha_0, \delta_0) \cdot h_a + \frac{\partial \phi}{\partial \delta}(y_0, \alpha_0, \delta_0) \cdot h_d,$$

With this we can impose conditions on the “future” of the orbit and find orientations that fulfil them (or show that the condition is unattainable). We try to find the angles α and δ that bring the trajectory where we want (close to the stable manifold of the target equilibrium point.)

Conclusions & Future Work

Conclusions

- We have understood the linear dynamics around an unstable equilibrium point and how it varies when the sail orientation changes.
- We have designed station keeping strategies using these information and applied them to a particular mission.
- We have discussed the robustness of these algorithms when different fonts of errors are included.
- We have seen that the countability of the sail is strictly related to the nature of the neighbourhood of the equilibrium point we want to keep the sail close to.

Future Work

- Test the robustness of these station keeping strategy around equilibrium points on a more realistic model (i.e. including the gravitational attraction of the other planets, variations on the solar radiation pressure, etc).
- Derive new station keeping strategies using the information given by the variational equations of the flow to derive a sequence of changes for the sail orientation.
- Extend the ideas station keeping strategies around equilibrium points to maintain the trajectory of a solar sail close to an unstable periodic orbits.

Merci de votre attention